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Christian Dudel[1]

A Nonparametric Partially Identified Estimator for Equivalence Scales

Abstract

Methods for estimating equivalence scales usually rely on rather strong identifying assumptions. This paper considers a partially identified estimator for equivalence scales derived from the potential outcomes framework and using nonparametric methods for estimation, which requires only mild assumptions. Instead of point estimates, the method yields only lower and upper bounds of equivalence scales. Results of an analysis using German expenditure data show that the range implied by these bounds is rather wide, but can be reduced using additional covariates.

JEL Classification: C14, D12

Keywords: household equivalence scale; partial identification; matching estimator

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1 Introduction

Household equivalence scales are routinely applied in research on poverty and inequality. They are used to adjust household income (or expenditure) of households of different size and composition. The resulting equivalized income is assumed to be directly comparable across households. More specifically, equivalence scales indicate how much more income a household of type a needs to reach the same welfare level as a reference household of type b. For example, using a household of a childless couple as a reference, a value of 1.3 for a household consisting of a couple with one child would mean that the latter household needs 1.3 times as much income to reach the same welfare level as the former household.

Many different methods for estimating household equivalence scales have been proposed in the literature. For an overview see Coulter et al (1992). In many cases, these methods rely on rather strong assumptions regarding household behavior and parametric structure to identify equivalence scales. Furthermore, data requirements are often high. For example, methods based on demand systems are derived from assumptions on cost (or utility) functions and require price variation in the data.

Recently, Szulc (2009) suggested the use of matching estimators based on the potential outcomes framework introduced by Donald Rubin (Holland, 1986; Rubin, 2005), which is commonly used as a starting point for the econometric literature on policy evaluations and treatment effect estimation. Compared to other methods, this approach for equivalence scale estimation has the advantage of relatively small demand in terms of data and it can be seen as a non-parametric generalization of the classic approaches by Engel and Rothbarth (for a description of these, see e.g. Deaton and Muellbauer, 1986).

Most of the literature on treatment effect evaluation analyzes the effect of a specific treatment on an outcome of interest. More precisely, let C be an indicator variable which equals 1 if the treatment has been received and 0 otherwise; Y denotes the outcome. The basic reasoning of the potential outcomes framework is that for each unit i, there exist two potential outcomes (see, e.g., Holland, 1986): y_i^1 is the outcome given the treatment and y_i^0 is the outcome which would be realized in the absence of treatment. In many cases, analysis is concerned with the average treatment effect (ATE) $\mathrm{E}(Y^1 - Y^0)$. Because in practice either y_i^1 or y_i^0 can be observed and never both, certain assumptions and techniques are used to arrive at an estimate of the ATE (see, e.g., Imbens, 2004).

Adapting ideas to the case of household equivalence scales, for each household i and each of two possible household compositions $C = 0$ (e.g. childless couple) and $C = 1$ (e.g. couple with one child), there exist two pairs of potential outcomes: u_i^0 and $y_i^0(u_i^0)$ are the level of welfare respectively the income needed to reach this level given composition $c_i = 0$; u_i^1 and $y_i^1(u_i^1)$ are the welfare level and the income needed to reach this welfare level given composition $c_i = 1$. Here, $y_i^j(u_i^j)$ can be interpreted as the value of the cost function of household i with composition j and welfare level u_i^j.

A household-specific equivalence scale can then be defined as the ratio $y_i^1(u)/y_i^0(u)$ for some welfare level u. In practice, again only one of the pairs of potential outcomes is observed for each unit i. That is, only $y_i = c_i y_i^1(u_i^1) + [1 - c_i]y_i^0(u_i^0)$ and $u_i = c_i u_i^1 + [1 - c_i]u_i^0$ are known. Furthermore, even if both pairs of outcomes could be observed, this would not suffice to calculate the ratio given above, because only $y_i^1(u_i^1)/y_i^0(u_i^0)$ could be calculated and not $y_i^1(u_i^0)/y_i^0(u_i^0)$ or $y_i^1(u_i^1)/y_i^0(u_i^1)$.

The reasoning behind the proposal of Szulc (2009) is that for households of composition $C = 0$, $Y^0(U^0)$ is known. If U is a function $f(X)$ of an observed welfare indicator X, $Y^1(U^0) = Y^1(f[X])$ can be estimated through matching methods using households of composition $C = 1$ and the same value of X. For households of composition $C = 1$, $Y^1(U^1) = Y^1(f[X])$ is known and $Y^0(f[X])$ can be estimated from households of composition $C = 0$. Note, though, that matching does not identify the joint distribution of $Y^0(f[X])$ and $Y^1(f[X])$ (see e.g. Abbring and Heckman, 2007). Only the marginal distributions are known. Because of this, the expected value and distribution of the ratio Y^1/Y^0 are not identified. Szulc (2009) solves this problem by estimating the geometric mean of Y^1/Y^0, $\mathrm{G}(Y^1/Y^0)$. Once the basic setup has been introduced in section 2, this geometric mean matching estimator (GMME) and some of its properties will be discussed in section 3. In section 4 an alternative estimator is proposed which builds on recent developments in the treatment effect literature. This estimator is based on partial identification of $\mathrm{E}[Y^1/Y^0]$ and only gives lower and upper bounds instead of a point estimate. An empirical example is given in section 5. Section 6 concludes.

2 Setup

For each observation i of a sample of n units, c_i and y_i, as defined in the previous section, and $\mathbf{x}_i$ are observed. $\mathbf{x}_i$ is a vector of characteristics of household i which capture its welfare level. Household welfare is assumed to be a function $u_i = u(\mathbf{x}_i)$ of $\mathbf{x}_i$. More specifically, for two households i and j we require $u_i = u_j$ if $\mathbf{x}_i = \mathbf{x}_j$. That is, the welfare indicator(s) in $\mathbf{x}$ can be used to check whether two households have the same welfare level. Note that this implies that given $\mathbf{x}$, household composition C gives no additional information on household welfare, i.e. $u(\mathbf{x}_i, c_i) = u(\mathbf{x}_i)$.

Second, $u(\cdot)$ is assumed to be either strictly increasing or strictly decreasing in each of its arguments, i.e. either $\delta u(\mathbf{x})/\delta x_k > 0$ or $\delta u(\mathbf{x})/\delta x_k < 0$ holds, where x_k is the kth element of $\mathbf{x}$. If $u(\cdot)$ is strictly increasing and $\mathbf{x}$, $\mathbf{x}'$ and $\mathbf{x}''$ are equal apart from element k, for which $x_k < x'_k < x''_k$, it follows that $u(\mathbf{x}) < u(\mathbf{x}'') < u(\mathbf{x}'')$. In case of $u(\cdot)$ strictly decreasing, the last inequality is reversed. This assumptions allows us to order households and make statements of similarity of households, at least with respect to a single indicator x_k. Note that the theoretical derivations in the following sections generally only require the first assumption stated in the preceding paragraph, whereas estimation also requires the second assumption.

For example, $\mathbf{x}_i$ could include the expenditure share on food (Engel approach) or the expenditure on some good which is only consumed by adults (Rothbarth approach). Another possibility is the use of some measure of satisfaction with the financial situation of the household, which would correspond to the so called Leyden approach or subjective equivalence scales (see e.g. Van Praag and Van der Sar, 1988; Schwarze, 2003). Furthermore, $\mathbf{x}_i$ could include additional characteristics like age or education of household members. To keep notation simple, $\mathrm{E}(Y^j|U)$ will be used instead of $\mathrm{E}(Y^j(U)|U)$. Moreover, note that $\mathrm{E}(\mathrm{E}(Y^j|U)) = \mathrm{E}(\mathrm{E}(Y^j|\mathbf{x}))$.

3 The geometric mean matching estimator

The literature on treatment effect estimation establishes that if certain conditions are met, the average treatment effect $\tau = \mathrm{E}(Y^1 - Y^0)$ can be estimated through (see, e.g., Rosenbaum and

Rubin, 1983)

$$\mathrm{E}[\mathrm{E}(Y^1|C=1,\mathbf{x}) - \mathrm{E}(Y^0|C=0,\mathbf{x})]. \tag{1}$$

The first condition which needs to be met is called unconfoundedness and requires that the pair Y^1, Y^0 is independent of C given $\mathbf{x}$. This guarantees that

$$\mathrm{E}(Y^1|\mathbf{x}) = \mathrm{E}(Y^1|C=1,\mathbf{x}) \quad \text{and} \quad \mathrm{E}(Y^0|\mathbf{x}) = \mathrm{E}(Y^0|C=0,\mathbf{x}). \tag{2}$$

The second condition requires that $0 < \Pr(C=1|\mathbf{x}) < 1$ and guarantees that $\mathrm{E}(Y^1 - Y^0|\mathbf{x})$ is defined for all $\mathbf{x}$ (overlap condition). If both conditions are met, this is called "strongly ignorable treatment assignment" or "selection on observables" (Imbens, 2004). Additionally, the so called stable unit treatment value assumption is invoked which implies independence of the outcomes of observation i from treatment status of observation j.

Assuming the conditions stated above to hold, a simple estimator of the average treatment effect is given by

$$\hat{\tau} = \frac{1}{n}\sum_{i=1}^{n} \hat{y}_i^1 - \hat{y}_i^0, \tag{3}$$

where $\hat{y}_i^1$ and $\hat{y}_i^0$ are derived via nearest neighbor matching. Let $d(i,j)$ be a distance function defined on $\mathbf{x}$, which captures how similar two households i and j are. For example, Szulc (2009) follows Abadie and Imbens (2006) and uses $d(i,j) = ||\mathbf{x}_i - \mathbf{x}_j||_{\mathbf{V}}$ where $||\mathbf{a}||_{\mathbf{V}} = (\mathbf{a}'\mathbf{V}\mathbf{a})^{1/2}$ and $\mathbf{V}$ is a weighting matrix. For each household i of composition $c_i = 0$ $\hat{y}_i^0$ equals y_i. $\hat{y}_i^1$ is set to y_i^M which is given by the mean of $Y^1(U^0)$ of the m nearest neighbors of i of composition $c = 1$, which can be found through $d(i,j)$. That is, for each household of composition $c = 0$ households of composition $c = 1$ with similar welfare level are "matched" for estimation of $y_i^1(u_i^0)$. Households of composition $c = 1$ are treated in the same way, i.e. $\hat{y}_i^1$ equals y_i and $\hat{y}_i^0$ is found via matching.

Szulc (2009) uses $\ln \hat{y}_i^1$ and $\ln \hat{y}_i^0$ instead of $\hat{y}_i^1$ and $\hat{y}_i^0$ such that the estimator equals

$$\ln \tau_{GM} = \frac{1}{n}\sum_{i=1}^{n} \ln \hat{y}_i^1 - \ln \hat{y}_i^0 = \ln \mathrm{G}(\hat{Y}^1/\hat{Y}^0), \tag{4}$$

where G denotes the geometric mean and τ_{GM} the GMME. Taking the exponential gives the final result.

Although τ_{GM} can be easily calculated with standard software, it is well known that $\mathrm{G}(\hat{Y}^1/\hat{Y}^0)$ will always be smaller than $\mathrm{E}(\hat{Y}^1/\hat{Y}^0)$. More specifically, let σ^2 be the variance of $\hat{Y}^1/\hat{Y}^0$, $b > 0$ the upper limit of the support of the distribution of $\hat{Y}^1/\hat{Y}^0$, and $a > 0$ the lower limit. The difference between the expected value and the geometric mean will always be between $\sigma^2/(2b)$ and $\sigma^2/(2a)$ (see Cartwright and Field, 1978). That is, any increase in the variance (mean reverting spread) will decrease the geometric mean, which is a rather undesirable property.[1]

[1] Note that in practice the difference between geometric and arithmetic mean could be either large or rather small. For example, assume that interest lies with the comparison of childless couples and couples with one child. Assume further that $a = 1.1$ and that $b = 1.4$. For $\sigma = 0.05$ and $\sigma = 0.1$ the lower and upper bounds of the difference are $[0.018, 0.023]$ and $[0.036, 0.045]$, respectively. If we were to compare single person households to couples with $a = 1.2$ and $b = 1.8$, the bounds given $\sigma = 0.1$ are $[0.028, 0.041]$ and given $\sigma = 0.2$ they equal $[0.056, 0.083]$.

4 A partially identified estimator

An alternative to the GMME starts from $\mathrm{E}(Y^1/Y^0)$. This expected value can be written as

$$\mathrm{E}\left[\mathrm{E}\left(\frac{Y^1}{Y^0}\middle|\mathbf{x}\right)\right] = \mathrm{E}\left[\frac{\mathrm{E}(Y^1|\mathbf{x})}{\mathrm{E}(Y^0|\mathbf{x})}\right] - \mathrm{E}\left[\frac{1}{\mathrm{E}(Y^0|\mathbf{x})}\mathrm{Cov}\left(\frac{Y^1}{Y^0}, Y^0\middle|\mathbf{x}\right)\right]. \tag{5}$$

This decomposition clearly shows which quantities can be directly derived given the assumptions stated in the previous section. Given unconfoundedness the conditional expectations $\mathrm{E}(Y^1|\mathbf{x})$ and $\mathrm{E}(Y^0|\mathbf{x})$ on the right hand side can easily be estimated. The assumptions do not suffice to estimate the covariance of Y^1/Y^0 and Y^0, though. This is because the joint distribution of Y^1/Y^0 and Y^0 is not point identified. Only the marginal distributions of Y^1 and Y^0 are (see Fan and Zhu, 2009; Firpo, 2005).[2]

Höffding (1940) and Fréchet (1951) established that these marginal distributions can be used to derive bounds on the joint distribution. Let $F_1(y^1|\mathbf{x})$ and $F_0(y^0|\mathbf{x})$ be the conditional cumulative distribution functions (CDF) of Y^1 and Y^0, respectively. These can be estimated through $F_1(y^1|C=1,\mathbf{x})$ and $F_0(y^0|C=0,\mathbf{x})$. $F(y^1,y^0|\mathbf{x})$ denotes the joint conditional CDF. This joint conditional CDF is bounded by (see also Abbring and Heckman, 2007)

$$\max[F_1(y^1|\mathbf{x}) + F_0(y^0|\mathbf{x}) - 1, 0] \leq F(y^1, y^0|\mathbf{x}) \leq \min[F_1(y^1|\mathbf{x}), F_0(y^0|\mathbf{x})]. \tag{6}$$

Let $k(y^1, y^0)$ be a strictly superadditive (strictly quasi-monotone) function of y^1 and y^0.[3] Following Cambanis et al (1976), Fan and Zhu (2009) showed that bounds on $\mathrm{E}[k(y^1,y^0)|\mathbf{x}]$ can be obtained from the upper and lower bounds on $F(y^1,y^0|\mathbf{x})$. Let $\beta^L(\mathbf{x})$ and $\beta^U(\mathbf{x})$ denote the lower and upper bound of $\mathrm{E}[k(y^1,y^0)|\mathbf{x}]$, respectively. These bounds can be calculated by

$$\beta^L(\mathbf{x}) = \int_0^1 k\left(F_1^{-1}(t|\mathbf{x}), F_0^{-1}(1-t|\mathbf{x})\right) dt \tag{7}$$

and

$$\beta^U(\mathbf{x}) = \int_0^1 k\left(F_1^{-1}(t|\mathbf{x}), F_0^{-1}(t|\mathbf{x})\right) dt, \tag{8}$$

where $F_1^{-1}(u)$ and $F_0^{-1}(u)$ are the quantile functions of the marginal distributions of Y^1 and Y^0, respectively. If k is a subadditive function, bounds are reversed, so that (7) gives the upper and (8) the lower bound.

Note that the results of Cambanis et al (1976) only require k to be strictly superadditive (or strictly subadditive) on the support of Y^1 and Y^0. If we assume that $Y^1 > 0$ and $Y^0 > 0$, which is reasonable for income, $k(y^1,y^0) = (y^1/y^0 - \mathrm{E}[Y^1/Y^0])\,(y^0 - \mathrm{E}(Y^0))$ can be easily shown to be superadditive, and the result given above can be used to derive bounds on the covariance of Y^1/Y^0 and Y^0. Furthermore, $k(y^1,y^0) = y^1/y^0$ is subadditive and the (conditional) expectation

[2] The GMME uses the fact that if the marginal distributions of Y^1 and Y^0 are known so are the marginal distributions of $\ln Y^1$ and $\ln Y^0$. The three assumptions introduced in the preceding section carry over to this transformations as well. As a result, $\mathrm{E}(\ln Y^1 - \ln Y^0)$ is identified.

[3] A function $k(a,b)$ is said to be strictly superadditive if for $a_1 > a_0$ and $b_1 > b_0$ the following inequality holds:

$$k(a_1,b_1) + k(a_0,b_0) > k(a_1,b_0) + k(a_0,b_1)$$

It is said to be strictly subadditive (strictly quasi-antitone) if the inequality is reversed, i.e. if $-k$ is superadditive.

can be calculated directly. Nevertheless, in what follows (5) will be estimated making use of the covariance of Y^1/Y^0 and Y^0. This means that the first and the second term on the right hand side of (5) will be calculated separately, because the first term can be seen as a naive estimate and the second term as a correction term. The former is an interesting benchmark for results of other methods and results found in the literature.

Let β^L and β^U denote the lower and upper bound of the partially identified estimator for some quantity like the covariance of Y^1/Y^0 and Y^0. A plug-in estimator is given by

$$\hat{\beta}^L = \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}^L(\mathbf{x}_i) \quad \text{and} \quad \hat{\beta}^U = \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}^U(\mathbf{x}_i), \tag{9}$$

where $\hat{\beta}^L(\mathbf{x}_i)$ and $\hat{\beta}^U(\mathbf{x}_i)$ are derived through (7) and (8), respectively (see Fan and Zhu, 2009). To estimate (7) and (8), kernel estimators for F_j, $j = 1, 0$, are used as proposed by Fan and Zhu (2009):

$$\hat{F}_j(y|\mathbf{x}) = \frac{\sum_{i=1}^{n} I(y_i^j \leq y) I(c_i = j) K(\mathbf{x}_i - \mathbf{x})}{\sum_{i=1}^{n} I(c_i = j) K(\mathbf{x}_i - \mathbf{x})}, \tag{10}$$

where $I(\cdot)$ is the indicator function and $K(\cdot)$ is a multivariate kernel density function.

Estimation of bounds for equivalence scales proceeds in the following fashion. Let τ_{PI} denote equation (5) and $\hat{\tau}^L_{\text{PI}}$ and $\hat{\tau}^U_{\text{PI}}$ the estimates of its lower and upper bound. The latter can be estimated through

$$\hat{\tau}^L_{\text{PI}} = \frac{1}{n}\sum_{i=1}^{n}\frac{\hat{\beta}_{Y^1}(\mathbf{x}_i)}{\hat{\beta}_{Y^0}(\mathbf{x}_i)} - \frac{1}{\hat{\beta}_{Y^0}(\mathbf{x}_i)}\hat{\beta}^L_{\text{Cov}}(\mathbf{x}_i) \tag{11}$$

and

$$\hat{\tau}^U_{\text{PI}} = \frac{1}{n}\sum_{i=1}^{n}\frac{\hat{\beta}_{Y^1}(\mathbf{x}_i)}{\hat{\beta}_{Y^0}(\mathbf{x}_i)} - \frac{1}{\hat{\beta}_{Y^0}(\mathbf{x}_i)}\hat{\beta}^U_{\text{Cov}}(\mathbf{x}_i), \tag{12}$$

where $\hat{\beta}_{Y^j}(\mathbf{x})$ is a nonparametric estimate calculated as

$$\hat{\beta}_{Y^j}(\mathbf{x}) = \mathrm{E}(Y^j|\mathbf{x}) = \frac{\sum_{i=1}^{n} y_i I(c_i = j) K(\mathbf{x}_i - \mathbf{x})}{\sum_{i=1}^{n} I(c_i = j) K(\mathbf{x}_i - \mathbf{x})}, \tag{13}$$

where $K(\cdot)$ again is a multivariate kernel density function. $\hat{\beta}^L_{\text{Cov}}(\mathbf{x})$ and $\hat{\beta}^U_{\text{Cov}}(\mathbf{x})$ are estimated via (7) and (8).

5 Empirical example

The partially identified estimator was applied to data of the German Income and Expenditure Survey 2008 ("*Einkommens- und Verbrauchsstichprobe*", EVS). Equivalence scales were estimated for couples with one child (less than 14 years of age) using childless couples as reference and net household income as outcome. The following welfare indicators were used: expenditure share for food (Engel method); expenditure on clothing for adults (Rothbarth method); and homeownership-status and housing space per household member (housing). All welfare indicators were used seperately and in combination. Additional household characteristics included in the analysis cover the following: age of household head; education of household head; region of household (West or East Germany); and whether both partners are employed (dual-earner household).

Households with one or both partners above age 65 were excluded from the analysis, as well as households with no employed household members and households where at least one household member received unemployment benefits (for a discussion of the reasoning behind such data preprocessing see Dudel et al, 2014). Analysis was carried out with data on 7116 childless couples and 2249 couples with one child.

For estimation of the conditional CDF $F_j(y|\mathbf{x})$ and the conditional expectation $\mathrm{E}(Y^j|\mathbf{x})$ the methods for nonparametric estimation and bandwidth selection proposed by Hall et al (2004) and Racine and Li (2004) were applied. These are based on the product of individual kernels for the elements x_k of $\mathbf{x}$,

$$K(\mathbf{x}_i - \mathbf{x}) = \prod_{k=1}^{m_1} \frac{1}{h_k} K_k\left(x_{ik} - x_k\right) \prod_{l=m_1+1}^{m} K_l\left(x_{il} - x_l\right), \tag{14}$$

where m is the number of elements of $\mathbf{x}$, with the first m_1 elements continuous and the other $m - m_1$ elements discrete, and h_k is the bandwidth of the kernel function $K_k(\cdot)$ for variable x_k. The kernel function $K(\cdot)$ depends on the type of variable x. For continuous variables, a second-order Gaussian Kernel was used and for the discrete case the kernel function proposed by Aitchison and Aitken was utilized (see Hayfield and Racine, 2008). Calculations were done using the freely available statistical package R (R Core Team, 2014) and the `np` package by Hayfield and Racine (2008).

In addition to the partially identified estimator, the GMME was calculated following the steps outlined by Szulc (2009).[4] One-to-one matching was applied, i.e., for each household the nearest neighbor was used as a match. The weighting matrix $\mathbf{V}$ was specified as the inverse of the diagonal variance matrix of the variables. Furthermore, an regression-based approach for bias correction was employed (for details see Szulc, 2009; Abadie and Imbens, 2011). Calculations were performed using the `Matching` package for R provided by Sekhon (2011).

Results of both methods are given in table 1 and table 2. Table 1 includes results which only control for the welfare indicators and ignore further demographic variables like age or education, whereas the results shown in table 2 cover both welfare indicators and demographic variables.

Without controlling for demographic variables bounds are rather wide. For instance, the difference between upper and lower bound amounts to 0.45 in case of the expenditure share of food. If one assumes that 1.5 is a priori a plausible upper bound for the equivalence scale for couples with children compared to childless couples, the upper bounds in table 1 are not informative. In all cases the GMME attains values which are close to the lower bound. Because both lower and upper bound of the second term in (5) are negative, this means that the GMME is relatively close to a naive estimate which uses only the first term in (5) and ignores the covariance of Y^1/Y^0 and Y^0.

Results based on analysis controlling for demographics, as shown in table 2, differ somewhat. First, in all cases the difference between lower and upper bound is smaller than for the results in table 1. This means that adding informative variables allows to reduce the range of bounds, as

[4] Note that Szulc (2009) estimated the average treatment effect for the treated (ATET) which is defined as $\mathrm{E}(Y^1 - Y^0|C = 1)$ and differs from the average treatment effect $\mathrm{E}(Y^1 - Y^0)$ (ATE) which is considered here. For the current application, differences in results are negligible, though.

Table 1: Results without demographic variables

Welfare indicator(s)	Lower bound	Upper bound	GMME
Expenditure share food	1.09	1.45	1.09
Expenditure clothing/adults	1.05	1.53	1.06
Housing	1.16	1.61	1.18
All	1.18	1.51	1.19

Table 2: Results with demographic variables

Welfare indicator(s)	Lower bound	Upper bound	GMME
Expenditure share food	1.14	1.47	1.12
Expenditure clothing/adults	1.04	1.45	1.04
Housing	1.12	1.50	1.12
All	1.19	1.50	1.21

noted by Fan and Zhu (2009), though the effect is rather small. Upper bounds are more plausible and do not exceed 1.5 or only at a small margin. In case of the expenditure share for food and the combination of all welfare indicators the lower bound is larger, for the other two indicators it is smaller than the bounds in table 1. Again, the GMME is close to the lower bound in all cases.

Results for equivalence scales for Germany taken from the literature generally tend to be close to the lower bounds. The modified OECD scale attaches a scale value of 1.2 to couples with one child. Another example are the results given by Schwarze (2003), which range from 1.10 to 1.15.

6 Concluding remarks

In this paper a method for estimation of equivalence scales was proposed which relies on partial identification and nonparametric estimators and only requires rather mild assumptions as compared to other approaches found in the literature. Furthermore, the approach is flexible in that it can be used with any combination of welfare indicators. In this paper, expenditure data and data on housing were used, but other indicators like, for instance, satisfaction with the financial situation of the household could be used in addition or instead. Results show that without strong assumptions, the range of possible values as indicated by lower and upper bound tends to be large. Using more variables and thus more information allows to reduce the range of bounds, though.

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