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Anna Bohnstedt[1]

Are Public and Private R&D Investments Complements or Substitutes?

Abstract

We develop a general equilibrium model with heterogeneous firms à la Melitz (2003), where both the government and firms can invest into R&D to improve the country's technological potential. A higher technological potential raises the average productivity of firms, thus implying lower consumer prices, and eventually leads to a welfare gain. The government's public and firms' private investments are modelled in a three-stage game, in which the government in the first stage invests into a basic research level, and then firms conduct private R&D building on this publicly provided "technology" in the second stage. We find that private R&D investments are hump-shaped with respect to the basic research level. For lower levels public and private investments are complements, while for higher levels they are substitutes.

JEL Classification: O3, H4

Keywords: Heterogeneous firms; public and private R&D investments; basic research; innovation

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1 Introduction

Innovation is an essential ingredient of present and future growth, and significantly contributes to the achievement of economic and social objectives. Consequently, countries considerably engage in innovation activities. Table 1 presents the research and development (R&D) intensity, measured by the gross domestic expenditures on research and development (GERD) as a percentage of the country's gross domestic product (GDP), for 12 selected OECD countries and two years, 1999 and 2009. Although innovation today entails a much wider range of inputs than just investments into R&D,[1] this spending item is nevertheless a commonly used measure for a country's involvement in innovation activities (see OECD (2011a)).

Table 1 here

As can be seen, the R&D intensity of the 12 countries has increased over time, which highlights the importance of R&D investments as a source of sustained economic growth in modern economies. The OECD identifies four sectors of potential R&D contributors: The business sector and the higher education sector as well as the government and non-profit organizations. Table 2 depicts the percentage of GERD for each performing sector. The business sector thereby takes the leading position with an average of 67.35 % of GERD.

Table 2 here

Besides private R&D investments by the business sector, also public investments play a crucial role. In this context, especially the relationship between public and private investments into R&D is an extensively discussed issue in politics as well as in the literature. In particular, the focus of the discussion is on the question whether public and private R&D are complements or substitutes, i.e., whether public R&D reinforces or crowds out private R&D activities.

In case of complementarity public R&D stimulates or supports private R&D. Public R&D investments encourage firms to take part in R&D projects, which would not have been interesting to private investors without public support.[2] These are projects, which are too risky, too expensive or have too little short-term economic outcome for firms to conduct private R&D like e.g. in the field of health, environment or national defence.[3] If public and private R&D are substitutes, however, then public R&D is assumed to reduce private R&D activities. Since necessary research is already done by the government, firms are not any longer interested to engange in private R&D. As table 2 shows, governments as well as firms vitally invest into R&D. However, from that data we can not draw any conclusions regarding the relationship between public and

[1]See Takalo (2012) for a survey on public innovation policies.

[2]See e.g. the huge literature that discusses the under-investment problem with respect to private R&D due to market failures, i.e., private investments into R&D are lower than it would be socially optimal.

[3]See OECD (2011d) for selected examples for direct and indirect public support to business R&D in OECD countries.

private R&D. Considering the empirical literature on this topic, results are mixed (see among others García-Quevedo (2004), Capron and van Pottelsberghe (1997)) with slightly more studies finding a positive and complementary relationship between public and private investments into R&D (see among others González and Pazó (2008), Almus and Czarnitzki (2003), Czarnitzki and Fier (2002), Guellec and van Pottelsberghe (2000), Lach (2000), and David et al. (2000) for a survey). However, contrary to the complementarity hypothesis, e.g. Wallsten (2000) finds that public R&D investments and private R&D investments are rather substitutes.

In order to shed light on the controversial addressed question if public and private research investments are complements or substitutes, we develop a theoretical model where both the government and firms invest into R&D. While there is a large empirical literature on this topic (see above), the theoretical literature here is way smaller. The seminal contributions by Arrow (1962) and Nelson (1959) help us to understand the basic benefits of public support to private R&D if private R&D is lower than it would be desirable. However, to the best of our knowledge, there is no theoretical model that formally studies the relationship between public and private R&D investments addressing the complements versus substitutes question. We do so in an recent heterogeneous firms framework à la Melitz (2003). Firm heterogeneity is by now a well-established empirical fact (see among others Bernard and Jensen (1999), Aw et al. (2000), Clerides et al. (1998)), and starting from the seminal model by Melitz (2003) a large theoretical literature, which accounts for that fact, has developed.

In our framework of heterogeneous firms in a closed economy, entrepreneurs pay a sunk entry cost before they randomly draw their productivity level from a known distribution. As in Bohnstedt et al. (2011), we allow for endogenous productivity improvement. Investments into R&D increase the country's technological potential, which is modelled as a right shift of the productivity distribution entering firms draw from.[4] However, while in Bohnstedt et al. (2011) only public R&D investments are taken into account, this paper also considers private R&D investments conducted by firms. We model the government's public and firms' private R&D investments as a three-stage game. In the first stage the government invests into a basic research level, which could also be interpreted as some basic "technology". The government's motive to invest into R&D is that an increase in the technological potential yields a higher average firm productivity, which implies lower consumer prices, and eventually leads to a welfare gain. In the second stage firms then conduct private R&D investments building on this publicly provided "technology". Private investments also increase the country's technological potential. This increases the firms' expected profits, which represents the firms' motive to invest. The third stage derives the equilibrium. Contrary to what may seem obvious at first, we find that higher public R&D investments, and hence a higher level of basic research, do not necessarily stimulate firms' private investments. We rather show that there exists a hump-shaped relationship between public and private R&D investments in equilibrium. For lower levels of basic research public

[4] This is in spirit of Demidova (2008). She models exogenously given productivity differences across countries by assuming distributions that are ordered in terms of hazard rate stochastic dominance. In our model an increase in the technological potential generates an improved productivity distribution that first stochastically dominates a productivity distribution with a lower technological potential.

and private research investments are complements, while for higher levels they are substitutes. Intuitively, we can distinguish two opposing effects. For lower levels the "public R&D effect" dominates. Public R&D investments offer firms a better "investment technology", which increases the value of entry and thus stimulates private R&D investments. For higher levels, however, we find a R&D induced "competition effect", which counteracts the "public R&D effect". The increased toughness of competition in the industry makes private R&D investments less profitable since the value of entry decreases.

The rest of the paper is organized as follows. Section 2 presents the setup of the model. In section 3 we introduce the government's public and firms' private R&D investments as a three-stage game into the model and present our results. Section 4 concludes.

2 Closed economy

Our setup closely follows the one by Pflueger and Suedekum (2013). The closed economy is populated by L workers who inelastically supply one unit of labor each. Labor is the only factor of production and perfectly mobile across two industries: A homogeneous good sector A with constant returns to scale and perfect competition and a manufacturing sector C with increasing returns to scale and monopolistic competition. In sector C firms are heterogeneous with respect to their productivity, and each firm produces one single variety out of a continuum of differentiated varieties.

2.1 Preferences

Preferences of a representative household h are defined over the homogeneous good and the set of differentiated varieties Ω. Utility is given by the following quasi-linear, logarithmic function with constant elasticity of substitution (CES) subutility over the set of varieties

$$U = \beta \ln C_h + A_h \quad \text{with} \quad C_h = \left(\int_{z \in \Omega} q_h\left(z\right)^{\rho} \mathrm{d}z \right)^{1/\rho}, \tag{1}$$

where $0 < \rho < 1$ and $\beta > 0$, and where the household's consumption of a variety z is given by $q_h\left(z\right)$. The elasticity of substitution between any two varieties is given by $\sigma \equiv 1/\left(1-\rho\right)$. The CES price index reads as $P = \left(\int_{z \in \Omega} p\left(z\right)^{1-\sigma} \mathrm{d}z \right)^{1/(1-\sigma)}$. Utility maximization implies per-capita expenditures $PC_h = \beta$ for the manufacturing good and $A_h = y_h - \beta$ for the homogeneous good, where y_h is the household's income. We assume $\beta < y_h$ to ensure that the preference for varieties is not too large, and hence there is positive production of both sectors in equilibrium. Indirect utility is then given by

$$V_h = y_h - \beta \ln P + \beta \left(\ln \beta - 1\right). \tag{2}$$

We drop the subscript h from now on as all households are identical. Total demand for a single variety z is given by $q\left(z\right) = \beta L p\left(z\right)^{-\sigma} P^{\sigma-1}$, and revenue for a single variety z reads as $r\left(z\right) = p\left(z\right) q\left(z\right) = \beta L \left(P/p\left(z\right)\right)^{\sigma-1}$.

2.2 Production and firm behavior

In sector A one unit of labor is transformed into one unit of output. The homogeneous good is used as the numéraire. The price for that good is normalized to one, and since workers are mobile across sectors, the wage in the closed economy is also equal to one. In the manufacturing sector C, a firm needs $l = f + q/\varphi$ units of labor to produce q units of output, where f is the overhead cost and $1/\varphi$ represents marginal costs. While the overhead cost is the same for all firms, marginal costs are heterogeneous across firms. A higher value of φ implies lower marginal costs $1/\varphi$ and hence represents a higher firm-level productivity. Due to the monopolistic competition framework with a continuum of firms, every single firm has zero mass and thus take the price index P as given. Consumers have iso-elastic demands, and firms charge prices which are constant mark-ups over firm-specific marginal costs, $p(\varphi) = 1/(\rho\varphi)$. As firms differ only in productivity, total demand, revenue, and profits for a single variety can be rewritten as a function of φ: $q(\varphi) = \beta L (\rho\varphi)^{\sigma} P^{\sigma-1}$, $r(\varphi) = \beta L (\rho\varphi P)^{\sigma-1}$, and $\pi(\varphi) = r(\varphi)/\sigma - f$, respectively. It is easy to see that a firm with a higher productivity φ charges a lower price, sells a larger quantity, and has higher revenue and profits. The CES price index can be rewritten as follows

$$P = M^{1/(1-\sigma)} p(\widetilde{\varphi}) = M^{1/(1-\sigma)} \frac{1}{\rho\widetilde{\varphi}} \quad \text{with} \quad \widetilde{\varphi} \equiv \left[\int_0^{\infty} \varphi^{\sigma-1} \mu(\varphi)\, \mathrm{d}\varphi\right]^{1/(\sigma-1)}, \tag{3}$$

where M is the mass of manufacturing firms (consumption variety), $\mu(\varphi)$ is the productivity distribution, and $\widetilde{\varphi}$ is the average productivity across firms in the market.

2.3 Entry, exit, and the technological potential

At each point of time a mass M^E of firms enter the manufacturing industry subject to a sunk entry costs f_e. After paying the entry costs, the entrants learn about their productivity level φ, which is drawn from a Pareto distribution: $G(\varphi) = 1 - \left(\varphi^{MIN}/\varphi\right)^k$, with density $g(\varphi) = k\left(\varphi^{MIN}\right)^k \varphi^{-(k+1)}$, where $k > 1$ is the shape parameter and $\varphi^{MIN} > 0$ is the lower bound.[5] Figure 1 illustrates two Pareto distributions with different lower bounds $\varphi^{MIN}_{\text{high}}$ and $\varphi^{MIN}_{\text{low}}$. It is easy to see that with $\varphi^{MIN}_{\text{high}}$ the entire distribution is shifted to the right, which implies a higher expected productivity draw for firms. The productivity distribution with $\varphi^{MIN}_{\text{high}}$ has a first-order stochastic dominance over the productivity distribution with $\varphi^{MIN}_{\text{low}}$. As in Bohnstedt et al. (2011), we refer to the lower bound of the Pareto distribution as the country's *technological potential* in the following.

After learning about its specific productivity draw, every firm decides whether to start production and serve the market or to exit immediately. Active firms earn constant per-period profits as described above. With a too low productivity draw firms cannot cover per-period fixed costs. Therefore, those firms with a productivity draw below some threshold $\varphi < \varphi^*$

[5]This model setup follows Hopenhayn (1992) and Melitz (2003), and has become the seminal approach for studying firm heterogeneity in a general equilibrium context. Furthermore, the Pareto distribution is widely used in the literature on firm heterogeneity (see Bernard et al. (2003) or Melitz and Ottaviano (2008)), as it has convenient analytical properties and fits empirical firm size distributions fairly well (Axtell (2001)).

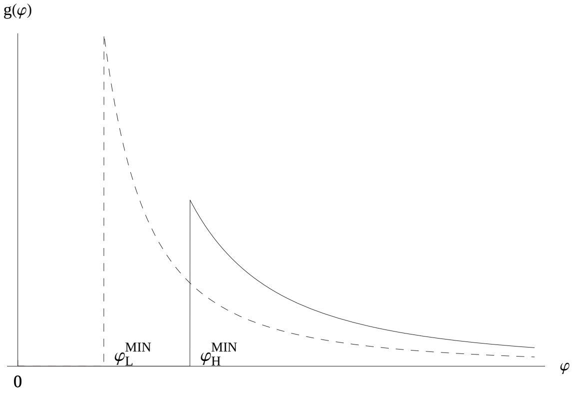

Figure 1: Different levels of φ^{MIN}

will exit, and those firms with a productivity draw above some threshold $\varphi > \varphi^*$ will stay in the market. As in Melitz (2003), every surviving firm can then be hit by a bad shock with probability $\delta > 0$ at each point of time, which is assumed to be uncorrelated with the firm's productivity draws. If this shock occurs, the firm must shut down. In the stationary equilibrium, the mass of firms which successfully enter the market equals the mass of firms which are forced to exit: $p_{in}M^E = \delta M$, where $p_{in} = 1 - G(\varphi^*)$ is the ex ante survival probability of entrants. The endogenous productivity distribution among surviving firms, $\mu(\varphi)$, is thus equal to $\mu(\varphi) = g(\varphi) / (1 - G(\varphi^*)) = k (\varphi^*)^k \varphi^{-(k+1)}$ if $\varphi > \varphi^*$ and equal to zero otherwise, i.e., it also follows a Pareto distribution with shape parameter k on the domain $[\varphi^*, \infty)$ with mean $\tilde{\varphi}$.

2.4 Equilibrium

As shown in Melitz (2003), two equations determine the closed economy equilibrium: The free entry condition (FEC) and the zero cutoff profit condition (ZCPC). The FEC states that firms will enter the industry as long as the value of entry, $v^E = E\left[\sum_{t=0}^{\infty} (1-\delta)^t \pi(\varphi)\right] - f_e$, is driven to zero. This in turn implies that the ex ante expected profit conditional on successful entry, $\bar{\pi} = \pi(\tilde{\varphi})$, is given by

$$\overline{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)} = \delta f_e \left(\frac{\varphi^*}{\varphi^{MIN}} \right)^k . \tag{FEC}$$

The ZCPC pins down the revenue of the cutoff firm, $r(\varphi^*) = \sigma f$, which by using $\overline{\pi} = r(\widetilde{\varphi})/\sigma - f$ and $r(\widetilde{\varphi})/r(\varphi^*) = (\widetilde{\varphi}/\varphi^*)^{\sigma-1}$ leads to

$$\overline{\pi} = f \left[\left(\frac{\widetilde{\varphi}}{\varphi^*} \right)^{\sigma-1} - 1 \right] = \frac{f(\sigma-1)}{k+1-\sigma}, \tag{ZCPC}$$

with $k > \sigma + 1$. Using (FEC) and (ZCPC), we solve for the equilibrium cutoff productivity denoted by

$$\varphi^* = \left[\frac{f(\sigma-1)}{\delta f_e (k+1-\sigma)}\right]^{\frac{1}{k}} \varphi^{MIN}, \tag{4}$$

where δf_e must be sufficiently low and/or f sufficiently high to ensure that $\varphi^* > 1$. Under the Pareto distribution, the average productivity among all active firms is then proportional to the cutoff productivity derived above: $\widetilde{\varphi} = \left(\frac{k}{k+1-\sigma}\right)^{1/(\sigma-1)} \varphi^*$. Furthermore, since aggregate expenditure on manufacturing goods βL must equal aggregate revenue of manufacturing firms $R = M\overline{r} = Mr(\widetilde{\varphi})$, we have $M = \beta L/\overline{r}$, where $\overline{r} = \sigma(\overline{\pi} + f)$. The mass of entrants is then $M^E = \delta M/(1 - G(\varphi^*))$, and the equilibrium masses of entrants and of surviving firms are given by

$$M = \left(\frac{k+1-\sigma}{\sigma k f}\right)\beta L \qquad \text{and} \qquad M^E = \left(\frac{\sigma-1}{\sigma k f_e}\right)\beta L. \tag{5}$$

Finally, using (2), (3), (4), and (5), indirect utility can be computed as follows

$$V = y + \beta \ln \varphi^* + \frac{\beta}{\sigma-1}\ln L + \kappa_1, \tag{6}$$

where $\kappa_1 = \beta(\ln(\beta\rho) - 1) + \beta/(\sigma-1)\ln(\beta/\sigma f)$ is a constant. Welfare is increasing in the population size L and in the cutoff productivity φ^*. Note from (4) that a higher technological potential φ^{MIN} increases the cutoff productivity, and hence leads to a welfare gain. The equilibrium masses of M^E and M, in contrast, are unaffected of φ^{MIN}. That is, in increase in the technological potential does not lead to more but to better firms in the long run equilibrium.[6]

3 Public and private research investments in the closed economy

We now consider a scenario where both firms and the government invest into R&D. A firm‘s private R&D investment is given by t_i, while the government's public R&D investments are denoted by T.

3.1 Stages of investments

We assume that the government first invests into a basic research level, and that firms can then conduct private R&D investments building on this basic research level or "technology" provided by the government. Formally, the stages can be summarized as follows:

1^{st} **Public R&D investments.** The government invests T into a country-specific basic research level denoted by $a(T)$. We assume that the government's investments have positive

[6] Note that in the short-run a higher technological potential rises the survival probability, and hence the firms' expected profits. This induces more entry, and we have a higher mass of firms in the market. More firms in the market, however, increase the toughness of competition, which causes exit of the least productive firms. This in turn increases the cutoff, lowers again the ex ante survival probability, the expected profits and, hence, the value of entry. Under the assumed Pareto distribution, and in a short-run perspective, these effects offset each other.

but decreasing marginal returns, and that the basic research level is zero if there are no investments. Formally, we have i.) $\partial a/\partial T \equiv a' > 0$, ii.) $\partial a'/\partial T \equiv a'' < 0$, and iii.) $a(T = 0) = 0$.

2^{nd} **Private R&D investments.** Before drawing from the productivity lottery, every firm i pays fixed entry costs f_e and invests t_i into R&D, which yields effective entry costs of

$$\hat{f}_e(t_i) = f_e + t_i. \tag{7}$$

A firm's private research investments t_i as well as the public basic research level $a(T)$ increase the effective technological potential for firm i, which is given by

$$\hat{\varphi}_i^{MIN} = \varphi^{MIN} \left[\ln\left(\mathrm{e} + a \cdot t_i\right)\right]^{\frac{1}{k}}. \tag{8}$$

Note that this functional form has the following properties. i.) Zero public or private investments yield $\hat{\varphi}_i^{MIN} = \varphi^{MIN}$, where φ^{MIN} is the country's initial technological potential. ii.) Given public investments, higher private investments increase the country's technological potential, i.e., $\partial \hat{\varphi}_i^{MIN}/\partial t_i = \hat{\varphi}_i^{MIN'} > 0$. iii.) Private investments have decreasing marginal returns, i.e., $\partial \hat{\varphi}_i^{MIN'}/\partial t_i = \hat{\varphi}_i^{MIN''} < 0$.[7]

3^{rd} **Equilibrium.** Given public and private R&D investments, entrants draw their productivity and the equilibrium is derived as described before.

In the following we solve this three-stage game via backward induction. The equilibrium stage, where the country's effective technological potential is already determined, is discussed in detail in section 2.4.

3.2 Private R&D investments

i.) **Firms' maximization problem.** Given the level of public R&D investments T, firms maximize their value of entry v^E with respect to private R&D investments. Doing so, every firm considers aggregate variables like average profits $\bar{\pi}$ and the cutoff productivity φ^* as given. Formally, firm i's maximization problem is given by

$$\max_{t_i} v^E = \left(\frac{\hat{\varphi}_i^{MIN}(t_i)}{\varphi^*}\right)^k \frac{\bar{\pi}}{\delta} - \hat{f}_e\left(t_i\right), \tag{9}$$

where $\bar{\pi}$ and φ^* are constants while $\hat{f}_e(t_i)$ and $\hat{\varphi}_i^{MIN}(t_i)$ as given by (7) and (8) are a function of t_i. The corresponding first-order-condition (FOC) can be written as

$$\frac{\partial v^E}{\partial t_i} = \frac{k\,\bar{\pi}}{\delta\,\varphi^*} \cdot \left(\frac{\hat{\varphi}_i^{MIN}}{\varphi^*}\right)^{k-1} \cdot \frac{\partial \hat{\varphi}_i^{MIN}}{\partial t_i} - \frac{\partial \hat{f}_e}{\partial t_i} = \frac{a \cdot \bar{\pi}}{\delta\left(\mathrm{e} + a \cdot t\right)} \left(\frac{\varphi^{MIN}}{\varphi^*}\right)^k - 1 = 0. \tag{10}$$

[7]This functional form allows for closed form solutions. Nevertheless, it is qualitatively possible to derive the key results without assuming explicit functional forms.

Note that the point of time of the private R&D investment decision is crucial. Firms invest into R&D before drawing from the productivity lottery. This is important because although firms will be heterogeneous after entry, when they have been assigned their individual productivity level, upon entry we have identical firms. Thus, in equilibrium, each firm will invest the same amount into R&D, i.e., $t_i = t$ for every firm i and hence effective entry costs are the same across firms, i.e., $\hat{f}_e(t_i) = \hat{f}_e(t)$ for every firm i.[8]

From the FOC as given by (10) we can disentangle two opposing effects of higher private R&D investments. On the one hand private R&D investments increase the value of entry due to an increase in the effective technological potential. On the other hand, they decrease the value of entry due to higher effective entry costs, which are constant and equal to one. The optimal level of private R&D investments, given public R&D investments and aggregate variables, satisfies $\partial v^E/\partial t = 0$ and is given by

$$\hat{t} = \frac{\bar{\pi}}{\delta} \cdot \left(\frac{\varphi^{MIN}}{\varphi^*}\right)^k - \frac{\mathrm{e}}{a}. \tag{11}$$

Note that the parameter a represents the basic research level or "technology" in the country, which is established by the government, and which positively depends on public R&D investments T. The expression in (11) is positive if a is sufficiently high. Intuitively, firms need to access a sufficiently good technology to transfer private investments into productivity gains.[9]

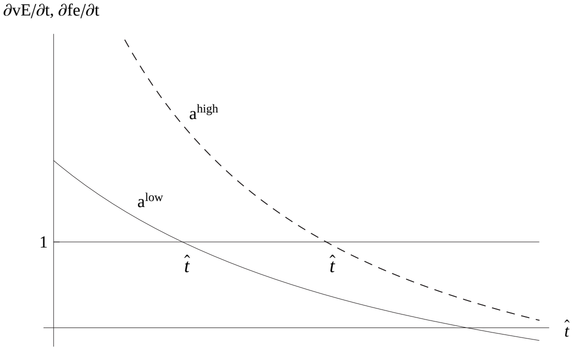

Figure 2: Optimal private R&D investments for different basic research levels

Figure 2 depicts the optimal investment level of private R&D for different basic research levels a. The optimum is given by the intersection of the horizontal curve, which represents constant marginal costs, and the positive but decreasing marginal gains in the value of entry of higher

[8]Regarding a potential free-rider problem, and to further clarify how private R&D investments effect the overall productivity distribution consider the following note. Imagine each firm invests into its "exclusive", firm-specific productivity lottery, which is Pareto distributed with shape parameter k and lower bound φ^{MIN} for each firm. A firm's R&D investments then increases the lower bound, and consequently the firm draws its productivity level from a "better" distribution than before, see 2.3. Since all firms are identical upon entry, they choose the same R&D investment level, and in the equilibrium the improved productivity distribution is the same for all firms. Furthermore, the parameters of the resulting mixture distribution are the same, too.

[9]A formal parameter restriction for the basic research level or "technology" a will be derived after the equilibrium is completely determined.

private R&D investments, which is the downward sloping curve. It can be easily seen that firms invest relatively more into private R&D if they have access to a better technology (dashed curve) than if they have only inferior technologies (solid curve).

Since $\hat{t}$ is the optimal level of private R&D investments *given* aggregate variables like the productivity cutoff and average profits, we can not yet conduct the complete comparative statics with respect to a. We first have to derive the equilibrium outcome.

ii.) **Aggregate outcome.** To close the second stage we use $\hat{t}$ in the FEC and ZCPC to determine the equilibrium, which then solely depends on public R&D investments T. The ZCPC remains unchanged while the FEC changes to

$$\bar{\pi}\left(t=\hat{t}\right)=\delta\cdot\hat{f}_e^*\cdot\left(\frac{\varphi^*}{\varphi^{MIN}}\right)^k \quad \text{with} \quad \hat{f}_e^*=\frac{1}{a}\cdot\exp\left\{1+W\left(\frac{af_e}{\mathrm{e}}-1\right)\right\}, \tag{FEC}$$

and where $W(\cdot)$ is the Lambert W function. We assume $a>\tilde{a}\equiv \mathrm{e}/f_e$ in the following to ensure that the argument of W is positive and hence we have $W(\cdot)>0$ and $W'(\cdot)>0$. We also know that $W(0)=0$ and that $W(\mathrm{e})=1$. For the properties of the Lambert W function also see Behrens et al. (2012). Setting ZCPC and FEC equal, we get the cutoff productivity in the second stage, which is given by

$$\varphi^*=\left[\frac{f(\sigma-1)}{\delta(1+k-\sigma)}\frac{1}{\hat{f}_e^*}\right]^{\frac{1}{k}}\varphi^{MIN}. \tag{12}$$

Finally, using (12) yields the equilibrium private R&D investment level, given the basic research level a

$$t^*=\frac{1}{a}\cdot\exp\left\{1+W\left(\frac{af_e}{\mathrm{e}}-1\right)\right\}-\frac{\mathrm{e}}{a}=\hat{f}_e^*-\frac{\mathrm{e}}{a}, \tag{13}$$

and where the aforementioned parameter restriction $a>\tilde{a}$ ensures that private R&D investments are positive.

We are now interested in the comparative static results with respect to the basic research level a. Recall that a is an endogenous variable, which positively depends on the amount of public R&D investments T. Since public investments into R&D are to be determined in the first stage (see section (3.3)), note that we conduct the comparative statics analysis for a given level of a at this point of time. After the model is closed, we can then give some further insights regarding the impact of exogenous variables on public R&D investments $a(T)$, and consequently on private R&D investments in the equilibrium t^*.

First, the toughness of competition in the industry, represented by the cutoff productivity as given by (12), increases in a. Formally,

$$\frac{\partial\varphi^*}{\partial a}=-\frac{\varphi^*}{k\left(\hat{f}_e^*\right)^{\frac{1+k}{k}}}\cdot\frac{\partial\hat{f}_e^*}{\partial a}>0 \quad \text{since} \quad \frac{\partial\hat{f}_e^*}{\partial a}=\frac{\mathrm{e}\left(1-\exp\left\{W\left(\frac{af_e}{\mathrm{e}}-1\right)\right\}\right)}{a^2\left(1+W\left(\frac{af_e}{\mathrm{e}}-1\right)\right)}<0. \tag{14}$$

Second, it follows from (5) that the mass of entrants increases in a while the mass of surviving firms is independent of a. Intuitively, in the equilibrium private R&D investments yield not more but better firms. This is in line with the fact that we observe an increased toughness of competition, see (14). Third, we analyse the impact of the basic research level on private R&D investments. Using $\partial \hat{f}_e^* / \partial a < 0$, it is easy to see that the equilibrium private investment level t^* is humped-shaped in a. We have

$$\frac{\partial t^*}{\partial a} = \underbrace{\frac{\partial \hat{f}_e^*}{\partial a}}_{\text{competition effect}} + \underbrace{\frac{\mathrm{e}}{a^2}}_{\text{public R\&D effect}}, \tag{15}$$

where $\partial t^* / \partial a > 0$ for $a < a^{crit}$ and $\partial t^* / \partial a < 0$ for $a > a^{crit}$, and where $a^{crit} > \tilde{a}$ is some threshold level of public R&D investments beyond which public and private R&D investments turn from complements to subsitutes. Intuitively, notice from (11) that a higher basic research level a increases the marginal gains while the marginal costs of investments are constant. This stimulates private investments, and we refer to this effect as "public R&D effect". However, a higher basic research level also heats up competition (higher mass of entrants and higher cutoff productivity), which makes private R&D investments less profitable since the value of entry decreases in the cutoff productivity. This can also be seen by (11), where a higher cutoff productivity decreases the private investment level, and we call this effect the "competition effect".

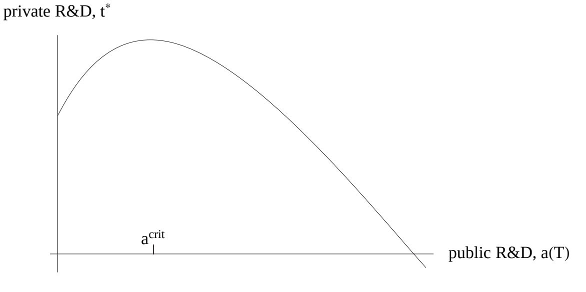

Figure 3: Hump-shaped relationship t^* and $a(T)$

Figure 3 illustrates the hump-shaped relationship between the equilibrium private R&D investments level t^* and the level of basic research $a(T)$. For low levels of a the public R&D effect, which stimulates investments, dominates. Public and private R&D investments are complements. For high levels of a the second effect of tougher competition outweighs the first effect and reduces the incentive for private investments, public and private R&D investments turn from complements to subtsitutes. We summarize the results of the second stage in Proposition 1.

Proposition 1: Private R&D investments. *i.) Private R&D investments are humped-shaped with respect to the basic research level. ii). Formally, private R&D and the level of basic research are complements for* $a < a^{crit}$ *and substitutes for* $a > a^{crit}$*, where* $a^{crit} > \tilde{a}$.

3.3 Public research investments

In the first stage we consider the government which levies a lump-sum tax from the households and spends the tax revenue on public R&D. The tax rate is denoted by x. Assuming $L = 1$, a balanced budget, and an efficient government, the tax revenue $L \cdot w \cdot x = T$ equals public investments into R&D, which eventually determine the basic research level in the country. The government maximizes welfare with respect to public R&D, and formally the government's maximization problem is given by

$$\max_{T} V = 1 - T + \beta \ln[\varphi^* (T)] + \kappa_1, \tag{16}$$

where φ^* is given by (12). The corresponding FOC can then be written as

$$\frac{\partial V}{\partial T} = -1 + \frac{\beta}{k} \frac{\left(\mathrm{e} - f_e a(T) + \mathrm{e}W\left(\frac{f_e a(T)}{\mathrm{e}} - 1\right)\right)}{\left(\mathrm{e} - f_e a(T)\right)\left(1 + W\left(\frac{f_e a(T)}{\mathrm{e}} - 1\right)\right)} \frac{a'(T)}{a(T)} = 0. \tag{17}$$

The FOC as given in (17) identifies the different effects of higher public investments into R&D. The first term represents welfare costs of investments, which are constant and equal to one. Intuitively, higher investments into R&D require higher lump-sum taxes, which reduce welfare. The second term represents welfare gains of investments. Numerical simulation shows that the term is positive and increasing in T. Higher public investments into R&D increase the basic research level in the country, and consequently the effective technological potential $\hat{\varphi}^{MIN}$. Considering these two effects, we have an unique and positive equilibrium level of public investments into R&D, T^*, and hence a unique a^*. Using a^* in t^* then closes the model.

Moreover, it directly follows from (17) that a higher preference for varieties β increases public investments. To develop economic intuition for this comparative static result, recall that we have a perfectly competitive outside sector. This leads to under-consumption of the manufacturing good due to mark-up pricing. Public R&D investments can then be thought of as a second-best policy to reduce this under-consumption problem since high productive firms set lower prices.[10] Regarding our comparative static analysis with respect to the basic research level a (see section 3.2), we can conclude that an exogenous increase (decrease) in the preference for varieties β increases (decreases) the equilibrium public R&D investments T^*, and consequently the level of basic research. However, the effect on private R&D investments t^* is unclear. A higher preference for varieties, and hence an increase in the basic research level, either yield an increase or a decrease in private R&D investments depending on whether public and private R&D investments are complements (increase) or substitutes (decrease).

Proposition 2: Public R&D investments. *There exists an unique and positive level of public R&D investments denoted by T^*, which maximizes welfare. ii) A higher preference for varieties β increases public R&D investments. iii) The effect of β on private R&D investments is unclear.*

[10] For the first-best policy in this type of model, see Pflueger and Suedekum (2013).

4 Conclusions

In this paper we develop a heterogeneous firms general equilibrium model à la Melitz (2003) that accounts for government's public and firm's private R&D investments in a three-stage game. In the first stage the government invests into R&D in order to establish a basic research level or basic "technology". In the second stage firms conduct private R&D investments building on the publicly provided "technology". The third stage derives the equilibrium. Public investments as well as private investments increase the country's technological potential, which is modelled as a right shift of the productivity distribution firms randomly draw from before they enter the market. Since an increase of the technological potential due to public and private R&D investments, implies a higher average productivity of firms in the country, R&D investments yield a welfare gain. With this model setup we address a broadly discussed issue in the literature as well as in politics, namely whether public and private R&D investments are complements or substitutes. Solving the three-stage game, we find a hump-shaped relationship between public and private R&D investments. For lower basic research levels public and private R&D are complements, while for higher levels they are substitutes. Intuitively, for lower basic research levels the "public R&D effect" dominates. Public investments stimulate or support private R&D investments by offering firms a better "investment technology". For higher levels the "competition effect" is the stronger one. Public R&D investments increase the toughness of competition in the market, which makes private R&D investments less profitable for firms.

Considering these findings, our model offers different potential implications for policy makers, and contributes to the above mentioned question whether, public and private R&D investments are complements or substitutes. First of all, our results confirm the common knowledge that innovation, measured by investments into R&D, is an important driver of growth and welfare. In our model the government's motive to conduct R&D investments is to maximize the country's welfare. Furthermore, we give a possible explanation for the mixed results in the empirical literature by analyzing the relationship between public and private R&D investments. Since we have a non-monotonic relationship, the answer to the question of complementarity versus substitution depends on the actual level of public R&D investments.

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Appendix

Table 1: GERD as a percentage of GDP in selected OECD countries

	R&D intensity	
	1999	2009
Finland	3.17	3.96
Sweden	3.56	3.62
Japan	3.02	3.33
Denmark	2.18	3.02
Switzerland	2.53	3.00
USA	2.64	2.79
Germany	2.40	2.78
Austria	1.90	2.75
Australia	1.47	2.21
France	2.16	2.21
Ireland	1.18	1.79
Norway	1.64	1.76

Source: OECD, Main Science and Technology Indicators Database, June 2011.

Table 2: GERD by performing sectors in selected OECD countries

	GERD by performing sectors, 2009 (as a percentage of GERD)			
	Business	Higher Edu.	Government	Non-profit
Finland	71.42	18.90	9.1	0.58
Sweden	70.40	25.09	4.44	0.08
Japan	75.76	13.41	9.21	1.61
Denmark	66.82	29.96	2.87	0.35
Switzerland	73.50	24.17	0.74	1.60
USA	72.60	12.85	10.60	3.94
Germany	67.55	17.55	14.90	-
Austria	70.56	23.84	5.43	0.25
Australia	60.77	24.21	12.33	2.68
France	61.91	20.55	16.35	1.19
Ireland	65.39	29.02	5.60	-
Norway	51.57	32.04	16.38	-
Average	67.35	22.63	8.99	1.36

Source: OECD, Main Science and Technology Indicators Database, May 2011.