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**Tail Parameters of Stable Distributions
Using One Million Observations of Real
Estate Returns from Five Continents**

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Michael Stein, Daniel Piazzolo, and Stoyan V. Stoyanov¹

Tail Parameters of Stable Distributions Using One Million Observations of Real Estate Returns from Five Continents

Abstract

This study focuses on global real estate return distributions. For our analysis, we employ the class of stable distributions that has become prominent in the real estate literature. We add to the literature by undertaking a global-scale analysis for the first time. By using data since the early 1990s, we show that there is considerable variation in the tail weights of return distributions, both between countries as well as among sectors within the countries. It is important to note that the tail parameters vary over time as well. Our results strengthen the recently discovered notion about non-constant tail parameters in stable distributions, which contradicts earlier findings about constant tail parameters. Additionally, we argue that merely changes over time were to be discovered, rather than pure methodological facts driving the variation, which is in contrast to the initial assumption associated with constant tail parameters. Our results provide an extensive overview of the tailedness of global real estate markets and offer a comprehensive insight into differing market distributions.

JEL Classification: G01, G10, G12

Keywords: Real Estate return distributions; stable distributions, tail dependence

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1 Introduction

The choice of the right distribution for financial assets has kept academics and practitioners busy since decades. Numerous studies have emerged in this area and technological progress has benefited the interested community in terms of applicability and feasibility of techniques. Although distribution-fitting is a well-researched topic, much remains to be discovered. Now, 50 years after Mandelbrot (1963) and Fama (1963) ¹ have shown that asset returns may deviate significantly from a normal distribution, researchers are still debating the type of distributional class or model that best describes asset returns. The discussion on asset returns naturally includes property returns, given real estate is the largest asset class. Nevertheless, it is a special case as the liquidity of direct real estate is not comparable to that of securitized assets like stocks and bonds. Young and Graff (1995), Young and Graff (1996), and Young and Brown (2012) provide excellent reviews on the evolution of distribution discussions and its relation with real estate research. Additionally, their seminal influence in financial research has thrown critical light on the adoption of both the Modern Portfolio Theory (MPT) and the Efficient Market Hypothesis (EMH). They have made the following observation:

MPT and EMH seem to have been introduced into real estate to justify the use of particular statistical techniques and portfolio strategies rather than as a consequence of empirical analysis of investment return and risk characteristics. In science, the situation is generally reversed: theories are developed to explain observations. (Source: Young and Graff (1996))

Especially Young and Graff (1995) explain the need to appropriately deal with the problematic aspects or strong assumptions underlying the MPT and EMH and how this relates to real estate.

The discussion surrounding the assumed distribution is generally a crucial part of any model. Numerous studies show that real estate returns, like many other asset class returns, are non-normally distributed. This renders application of theories like the mean-variance portfolio optimization to real world data problematic and further fuels the concerns discussed above. Among others, Liu et al (1992) and Myer and Webb (1993) first provided evidence for this. Using non-normality in their MPT application, Byrne and Lee (1997) show that the National Council of Real Estate Fiduciaries (NCREIF) data is non-normal.

¹ See Fama (1965a) and Fama (1965b) as well.

Benjamin et al (2001), Maurer et al (2004), and Coleman and Mansour (2005) also focus on the return (and risk) characteristics of real estate based on country-wise examinations.

It should be noted that the discussion on return distributions in finance is mostly related to securitized assets like stocks and bonds, which are mainly traded daily². Accordingly, examinations normally center around the distribution of returns over time. This is in contrast to studies in the (direct) real estate return domain, where mostly the cross-section of returns is examined and discussed.

Not only have researchers reported deviations from normality, studies aimed at conducting detailed examinations and estimations of real estate return distributions were carried out as well. Among such studies, investigations using stable Paretian, or (α)-stable distributions (stable distributions in the following), are most dominant in this field. Notably, Fama and Mandelbrot already turned the focus on stable distributions 50 years ago during the emergence of return distribution discussions. Nowadays, the use of stable distributions has become widely-accepted in real estate research with the initial application of the methodology to real estate return data by Young and Graff (1995). The following are most important aspects of stable distributions: (i) data have a stable distribution if a linear combination of two random selections of the dataset has the same distribution, location, and scale parameters, and (ii) data should be parameterized more flexibly than normal distributions. The normal distribution is a special case of the stable distribution and all other distributions may be well approximated by a stable distribution model, as reported in Rachev and Mittnik (2000) for example. The need for a better fitting model for financial data to represent the perceived departure from normality led to the usage of a more comprehensive class of distribution.

Furthermore, we use stable distributions as well and add to the literature in this field by examining a global dataset initially containing over one million observations. Like previous researchers, we find that the data is very well represented by the chosen stable distributions. Our findings remove the uncertainties surrounding time-variability, especially regarding the tail-weights of distributions. We find strong evidence that is in line with more recent findings by Richter et al (2011) and Young and Brown (2012), which are in contrast to the earlier notions on constant tailedness over time. Additionally, we present the findings on a global scale, where parameter results can be compared between sectors and countries. We find significant differences as a result of this exercise.

The paper is organized as follows. Stable distributions and related studies on real estate returns are discussed in the next section. In the third section, the real estate model is described, and data

² See McCulloch (1996), Rachev and Mittnik (2000), and Rachev (2003) among others.

and estimation methods are explained. Empirical results and implications are presented in the fourth section. The fifth section concludes and provides an outlook on future research.

2 Stable distributions and their use in real estate research

The use of stable distributions in real estate research goes back to Young and Graff (1995), who used stable distributions for the NCREIF database³. They found that the stable distributions capture the return structure much better than normal distributions. Follow-up studies in this area have been provided by Graff et al (1997), Young et al (2006), Young (2008), Richter et al (2011), and Young and Brown (2012), among others. Based on McCulloch (1986), studies until 2011 used quantile-regression approaches to estimate the parameters. All authors report that stable distributions fit the data very well. Interestingly, earlier studies concluded that the characteristic exponent—which defines the tail-weight of the distributions—is constant over time and across various types of property. This was contradicted later by Richter et al (2011) and Young and Brown (2012), where considerable variation in the characteristic exponent was reported over time. Results of the newer studies, which show differences over time, appear to be robust to the method of estimation: While the latter use maximum-likelihood estimation (MLE), the former use a quantile-regression approach. A third possibility is to estimate the parameters by using the characteristic function, as explained and compared in Rachev et al (2007).

Stable distributions in general can be best described by their characteristic function—the inverse Fourier transform of the probability density function. One of the most used parameterizations for a stable random variable X $S(\alpha, \beta, \sigma, \mu)$ ⁴ is found in Samorodnitsky and Taqqu (1994):

$$(1) \log(\phi_X(y)) = i\mu y - |\sigma y|^\alpha \left[1 - i\beta \operatorname{sign}(y) \tan\left(\frac{\pi\alpha}{2}\right) \right] \text{ for } \alpha \neq 1^5$$

where $\alpha, \beta, \sigma, \mu \in R$, $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$, $\sigma \geq 0$

$$\text{and } \operatorname{sign}(y) = \begin{cases} 1 & \text{if } y > 0 \\ 0 & \text{if } y = 0 \\ -1 & \text{if } y < 0. \end{cases}$$

³ Russell-NCREIF at that time.

⁴ Samorodnitsky and Taqqu (1994) provide the α -stable random variable notation $X \sim S_\alpha(\sigma, \beta, \mu)$ as well. We keep the initial notation with all parameters within brackets.

⁵ The function is discontinuous, with $i\mu y - |\sigma y| \left[1 - i\beta \left(\frac{2}{\pi}\right) \operatorname{sign}(y) \ln|y| \right]$ for $\alpha = 1$.

While Levy (1934) initially reported the stable class of probability distributions, Khintchine (1937) later demonstrated that stable distributions are among the class of infinitely divisible distributions. Unlike normal distributions that cannot exhibit heavy tails and skewness, stable distributions allow for a large variety of shapes and structures. The four parameters defining the shape of a stable distribution can be described as follows:

First, the characteristic exponent, α , called the index of stability or stable index, determines the weight of the distribution's tails. For values of α lower than 2, the shape of the distribution is more peaked at the location parameter and exhibits fatter tails. A parameter value of 2 corresponds to the tail index of a normal distribution (no mean exists for $\alpha < 1$). Second, the parameter β , which is bound between -1 (skewed to the left) and $+1$ (skewed to the right), determines the distribution's skewness and indicates whether the occurrence of returns is more probable for negative or positive realizations. Third, the parameter δ scales the distribution and is often seen as a generalized standard deviation. Fourth, as would be the case with any other commonly used distribution, the location parameter μ is responsible for shifting the distribution's peak to the left ($\mu < 0$) or the right ($\mu > 0$).

As mentioned above, the parameters could be estimated by using MLE, quantile-regression, or characteristic function approaches. While Young and Brown (2012) argue that the move to the MLE technique for estimation of parameters is the cause of the new observation of time-varying parameter estimates of the characteristic exponent α , the results of Richter et al (2011), who use the quantile approach, also suggest variability over time. Accordingly, albeit the choice of estimation technique may certainly affect the results, one may argue that the new findings are caused by other effects too. Evolvement of the industry over time and availability of larger time spans could be the cause of the new result of non-constant tail-weights over time. In fact, the estimation approach should lead to only marginally differing results anyway⁶. Additionally, all the results that imply that the tail-weight depends on the respective time period under consideration are well in line with theory. Given the property returns show sensitivity and variation to market movements, estimations of their return distributions should reflect the various cycles, crises, or boom and bust phases the markets undergo overtime.

Finally, fitted distributions can be evaluated by probability metrics, quantile-quantile (qq) plots or Kolmogorov-Smirnov tests. By comparing estimated and empirical distributions, one needs to verify that the resulting fitted distribution provides a reasonably good fit for the observed distribution. We

⁶ Hoehstoetter et al (2005) and Rachev et al (2007) compare the results obtained from applying all three different estimation techniques.

have shown graphical examples of several markets for various years in the empirical section. Furthermore, we report results from our chosen characteristic function estimation technique together with results of the two sample Kolmogorov-Smirnov tests for each estimation.

3 Data and Empirical Approach

3.1 The IPD database

IPD collects real estate data of directly held properties from large institutional investors. These investors supply such data in return for a portfolio analysis relative to the benchmark of all participants. However, no identifiable data on individual properties or individual funds are revealed. Calculation of market averages and benchmarks for countries, sectors, and segments (like “Office Barcelona”) is done for the public, since only the data supplier receives the information about the performance of his real estate portfolio relative to the benchmark. This system of communicating the results of portfolio evaluation confidentially to individual data suppliers ensures that they do not have an incentive to supply inaccurate data that does not fully reflect the true situation. Moreover, data suppliers pay IPD for examining and monitoring their relative performance in order to make necessary adjustments to the portfolio. Therefore, the dataset used should be free from reporting bias.

IPD publishes country indices for markets (and industrial, office, retail, residential, and other sectors) where various standards like sufficient database size, market coverage, regular valuation, and reliability of data are met. IPD receives data of about 70,000 properties from 1,500 funds across 30 countries spanning all five continents. For depicting our results in tables and figures, we have later grouped Australia and Asia together. These properties are valued at least once per year, and have a capital value of USD 1.4 trillion at the beginning of the year 2013.

The database, including the annual returns of property, was made available from 1990 onwards, with no exceptions regarding countries, sectors, or any other aspect. As the real estate return data is delivered to IPD on confidential basis, this is preserved by using anonymous datasets. Therefore, for each of the 1,043,972 observations of property obtained, the following seven-column data matrix was used: anonymous identifier number, year, country, sector, total return, income return, and capital gains.

3.2 The Real Estate Model

Young and Graff (1995) and other authors of studies mentioned in section 2 employ a real estate return model where the returns are simply explained by the specific sector. This means that for each property, p , the average return, μ_t , of the respective sector of properties, h , with the same usage is included to explain the variation in the return r of a given year t :

$$(2) \quad r_t(p) = \mu_t(h(p)) + \varepsilon_t(p)$$

The residual, or property-specific deviation, $\varepsilon_t(p)$, is assumed to be stable distributed. Moreover, the estimations are not based on the unadjusted property returns, but on the sector-corrected property returns. Although this is only a rough filtration of the returns, it fulfills the desired removal of structural effects in property returns. Regarding expected geographical influences, Young and Graff (1995) state the following:

Alternatively, we could have broken down the returns by major geographic region. We believe that property type, however, is the superior cut, because it is more likely that investment characteristics of commercial property differ for properties with different drivers of economic performance than that investment characteristics differ for properties with the same economic functional applications situated in different locales. The free flow of institutional real estate investment capital across the country—as contrasted with an earlier era in which capital and investment decisions were more local in nature—will tend to homogenize transient differences in investment characteristics across geographical regions for property of the same type.

While the authors above have examined returns of properties in the United States, our sample contains real estate returns from across the globe. This results in the possibility of huge differences in macroeconomic influences, state of the economy, country-specific effects, and points of the real estate cycle. Countries and/or their major real estate sub-markets may be in completely different phases, since direct contagion among real estate markets, if and when it exists, takes time to spread on a global scale. For example, one might expect the United States property returns in 2007/2008, especially in the residential sector, to be significantly different from the German residential property returns that year. Another example would be Spain since the government debt crisis in 2011 or bursting bubbles

and specific events in various markets in certain years.

To account for these possibly large differences, it is natural to follow the standard real estate model such that the return of each property in a given year is de-meaned not by the global average return in the respective sector, but the average return in the respective sector in the respective country, h_c :

$$(3) \quad r_t(p) = \mu_t(h_c(p)) + \varepsilon_t(p)$$

It follows that returns and errors are country-specific and corrected for the applicable average return in each country's sectors. This is accomplished by de-meaning each property return with the average of the respective sector in the relevant country, and fitting a stable distribution to all of the respective country's annual property returns in each sector under consideration. Since we apply the standard real estate model *within* each country, our approach is in line with previous studies. As data on total return, income return, and capital gain is available for each property, we can estimate whether each of these three property-specific variables follows a stable-distribution after being corrected for sector effects in the respective countries. As the IPD database was made available for the period 1990–2012, 23 years with five sectors (industry, office, retail, residential, and others) and three variables (total return, income return, and capital gain) were used. Depending on the starting point of data collection in various countries, this amounts to 345 distributions for each country in the database. Estimation was done only for combinations of country, sector, and year, where at least 100 observations were available, in order to have enough observations for proper parameter estimation. Naturally, this reduces the amount of distributions fitted, especially for smaller markets and earlier sample years. Aggregate results are not calculated as we are (i) most interested in variations over time, and between countries and sectors and (ii) aggregate results lead to multi-modal distributions in many cases⁷.

4 Empirical Results

4.1 Total Return Results

Estimation of parameters over the years 1993 to 2012 was in general very successful. The characteristic function method turned out to be highly efficient for estimation. There is absolutely no indication of poorer fits for markets containing, say 150 observations, when compared to fits done with 1,500 observations. Tables 1 to 5 report all parameter estimates of the total returns for 22 countries,

⁷ For example, if most European markets are in a different phase than Asian and American markets, one may end up with a mixture of three different stable distributions and three peaks.

which have the largest markets according to the IPD database. Due to space considerations, and since many countries did not contain enough observations in the earlier sample years, we report results from 2002 to 2012. All other results are available upon request

Each table is devoted to one of the sectors: industry, office, retail, residential, and other. As we are mainly interested in the characteristic exponent α and the scaling parameter σ , we report only those and the results of the two sample Kolmogorov-Smirnov tests⁸. Results for income return and capital gain estimation results are presented in Tables 6 to 15 in the appendix. Remaining parameter results, which have not been discussed, are available upon request.

In order to get an indication about how different the distributions may be, and to have an intuition for the parameter estimates, we show both empirical and estimated distributions for some selected office markets across different continents. Figure 1 depicts empirical and fitted distributions for the United Kingdom, Japan, United States, and South Africa for the six recent years. It is clear why stable distributions emerged as the preferred class to fit direct real estate returns.

Shapes of return distributions not only differ according to their location parameter and their width/scale, both of which could have been modeled with a normal distribution, they exhibit pronounced tailedness and skewness as well. Thus, the stable distributions with the four parameters are indeed needed. Furthermore, on comparing the empirical distributions with their estimated counterparts, one can see that estimation itself apparently serves us well in that the estimated distributions are very close to the empirical distributions, and provide reliable fits⁹. Apart from visual inspections, the two-sample Kolmogorov-Smirnov tests also indicate that the estimations fit the data well in the vast majority of cases. Additionally, some rejections of the null hypothesis are more of a location issue than caused by severe deviations in general. For example, the 2011 estimation for the UK has an over-estimation of the frequency of the location parameter, but still fits the data very well over the whole range. However, the fit appears to be a little more deviated in 2008. While this may explain rejections of the null hypothesis in the presence of generally well-fitted distributions, on some minor occasions a distribution with a parameter $\alpha < 1$ is not rejected. Both these types of observations may be a result of the slight sensitivity of the Kolmogorov-Smirnov test to the central region of a distribution.

Overall, the six example years of office property market returns in the countries with the largest real estate markets across four continents show that 20 out of 24 distributions are almost perfectly represented by the fitted stable distribution. With regard to the other countries, we can see from Table

⁸ We use the 95% confidence level to indicate whether the null hypothesis of identical distributions is rejected.

⁹ Checks using qq plots strengthen our assertion of a generally very good fit over a large set of sectors and countries.

1 that a vast majority of annual return sets are represented very well with the estimated parameters, even on grounds that the Kolmogorov-Smirnov test apparently often rejects the null hypothesis when the probability of the mean is overestimated. This result of very good fits holds for other sectors as well, as can be seen from Tables 2 to 5. Naturally, the other sectors have lesser results for interpretation, since majority of the property belongs to the office sector.

With a particular focus on the tail index, we turn to the estimates of the parameter α . Figures 2 and 3 show boxplots of the estimates of the characteristic exponent α from 2002 onwards. For the office and industrial sector in the four example countries, we can observe how the parameter estimates, as well as the bootstrapped range of estimates, emerge over time. The estimates reflect changes in the distributions and therefore, changing tailedness over time and across countries. For example, the over-heating in the US real estate markets from 2006 onwards led to considerably lower estimates of α , that is, higher tail-dependency. There is a lag in this development in the UK, which is in general accordance with market observations during the recent crisis. On the other hand, Japan shows a more resilient pattern over time, while tail dependence in returns for South African property appears to be close to that of a normal distribution. Time-variability is seen in most other areas and sectors as well. Table 4 presents observations from the residential sector. The US housing market peak and its subsequent break-down clearly demonstrates these effects in 2006 and 2007, with tail parameters of about 1.3, which are among the lowest estimates across countries and years.

The tailedness of markets appears to change according to market movements. A similar observation is noted for the scaling parameters. However, the interplay with the tail index is of much interest to us. For example, while the UK office market has scale parameters of 7 or higher for most years, on the contrary, Switzerland has scale parameters estimated to be above 3 only twice since 2002. While this is not surprising since Switzerland is known to be a very robust, conservative, and fairly closed real estate market, tail parameters well below those for the UK are observed. This might lead to concerns about the reliability of the estimates, especially since we only had 167 observations for Switzerland at the beginning of the reported time span (2002). However, discussions regarding the tail parameter and leptokurtotic distributions must be done after considering two important facts.

First, not only does the measure of kurtosis tell us about the tail weights of a distribution, but it is also a result of the curvature around the mean. Thus, very high concentration of probability at the peak and a steep decrease around it, in combination with high tail probabilities, may facilitate the low parameter estimate of α . Second, and more important, is the interplay with the scale parameter.

Figure 4 contains the empirical and fitted distribution of the office market total returns for UK and Switzerland in the year 2012. Although the characteristic exponent of Switzerland is 1.21 and that of UK is 1.61, the effect of the scale parameter (UK 6.16 against Switzerland 2.31) leads to relatively much higher probabilities with large deviations from the mean in case of the UK, compared to Switzerland. This is evidenced by the risk measure expected tail loss as well¹⁰. At the 95% confidence interval, the expected loss is -35.83% for UK and -27.04% for Switzerland. In conclusion, it can be noted that the results for Switzerland are in no way estimation errors, but a result of the curvature of the distribution. The notion of a relatively more solid market in Switzerland is not questioned, but strongly confirmed, by our results.

Overall, the stable distributions perform very well in explaining the distributions of total returns. For the year 2012, the null hypothesis is rejected only in three out of 22 cases. Consequently, in 86 percent of the examined countries, the test does not reject the assumption that the fitted and empirical distributions are the same. This coupled with the fact that apparently most rejections are the result of an overestimation of the mean, as seen for the UK office sector in three of the last six years, we conclude that the estimations provide highly reliable results for the parameters.

For the other sectors, the results are comparable with respect to variation over time and between countries. There are variations in parameter results between sectors within a country and year as well. However, while the parameters between sectors do not differ much in some years, there are large differences in other years. We consider this a reasonable result, as effects from financial market and macroeconomic developments may have varying strength in respective sectors. This depends on the country and year under consideration as well.

4.2 Further Examinations and Implications

With data on income returns and capital gains, one can examine whether the stable distributions fit the two components of the total return comparably well. From Tables 6 to 10, we can see that the income return distributions are fitted considerably close by the estimated stable distributions. However, capital gains do not appear to be equally well described by the stable distribution fits. This is apparent in Tables 11 to 15. From the estimation of parameters for income returns and capital gains, we obtain results that correspond to observations from the distribution fitting for total returns.

In this context, there is time variability and dispersion both between countries and sectors.

¹⁰ Expected tail loss (ETL) or conditional value at risk (CVaR) for continuous distributions is extensively discussed in Rockafellar and Uryasev (2002), whereas performance ratios in the context of asset allocation are broadly reviewed in Farinelli et al (2009) among others.

The result of the distributions that appear less leptokurtotic and therefore, somewhat higher estimates for the characteristic exponent α of income return distributions, is in line with the findings of Richter et al (2011) for Germany. They also report a more erratic behavior for the characteristic exponent in capital gains estimations. We can also make a general observation in this regard, although the overall fit of estimations is less precise than the counterparts for income and return. Regarding the relation of parameter estimates, we can see from Figure 5 that the characteristic exponent of total returns is more in line with that of capital gains than with that of income returns. This figure shows all the estimated characteristic exponents of the office sectors for the three datatypes. The scatter plots indicate that the tailedness of total returns is merely driven by the capital gain tailedness, than by large deviations in income returns.

All the results offer a clear answer to the question of whether the characteristic exponent is constant over time or not. We found time-varying estimates that are in line with Richter et al (2011) and Young and Brown (2012). While the latter attribute the change in perception of constant or non-constant estimates to the estimation technique used, the authors believe that, in the context of the implication on comparing all three studies, earlier results pertained to shorter time periods. A comparison of maximum likelihood estimates and quantile-based estimates in Young and Brown (2012) indeed shows time variation in both.

In general, our results are roughly in line with the findings of Richter et al (2011) and Young and Brown (2012) for Germany and the United States, respectively. It should be noted that IPD data has been utilized only in the former, while Young and Brown (2012) employed the NCREIF database. Thus, our analysis serves as an extension of previous studies on a global scale and answers the question on time-variability, cross-country dispersion, and sector differences of the tail-weight in return distributions.

It is interesting to analyze the differences between the various sub-sectors and countries. Figure 1 in particular reveals the diverse shape of the distributions. It is doubtful whether other distribution types would be capable of being fitted to these highly tail-dependent distributions. Moreover, tests for normal distribution of returns would make less sense for comparative reasons, given our results and because the normal distribution is a special case of the stable distribution—therefore, it could be obtained from our estimations anyway.

The discussion on characteristic exponents and scale parameters highlighted a significant aspect. While tail weight is a crucial factor when it comes to evaluation of the riskiness of markets, it is not the only important factor. Scale parameters determine the width of the distribution, and are a crucial determinant of return dispersion. In this context, the example of office return distributions in the UK and Switzerland is highly explanatory. Therefore, we should be cautious about judging market riskiness solely on the basis of the characteristic exponent.

5 Conclusion

The study clearly met the aim of estimating stable distributions for real estate returns towards answering the question regarding differences over time, and across countries and continents. Not only can variations over time be observed, but they are also in line with market movements and global developments. For example, the recent global financial crisis and the ensuing economic slump have an effect on parameter estimates, and as a result, more tailedness and higher dispersion is found for respective years and markets. Furthermore, differences with respect to riskiness and variation, as indicated by our results, are in line with observations from the past and the markets, as the comparison between UK and Switzerland exemplified.

Thus, our results provide an extensive overview of the tailedness of global real estate markets and offer the most comprehensive insight into differing distributions of markets. The comparability of our results to other country-wise examinations demonstrates the reliability of the results and the robustness of estimation methods used.

Although we are in line with other studies that analyze real estate return distributions for given time spans of calendar years for which the observations were made, we cannot rule out that the tail parameter variation and interplay with the scale parameter would change if time-series effects are taken into account. For example, if one would control for general autoregressive conditional heteroscedasticity (GARCH)-type behavior in the returns, the scale parameter might change along with the estimate of the characteristic exponent.

Future research could focus in detail on the respective parameters of markets, and the drivers of change over time and across markets. Promising research could be done in the field of portfolio management and risk management, by including stable distribution fitting. This relates to the already used standard real estate model as well. As the returns are solely corrected for sector effects, other effects may be controlled for, with the remaining residuals being in the focus of a stable distribution

fitting exercise. It is yet to be found how the distributions would be shaped then and whether they will still exhibit the characteristics that make stable distributions necessary.

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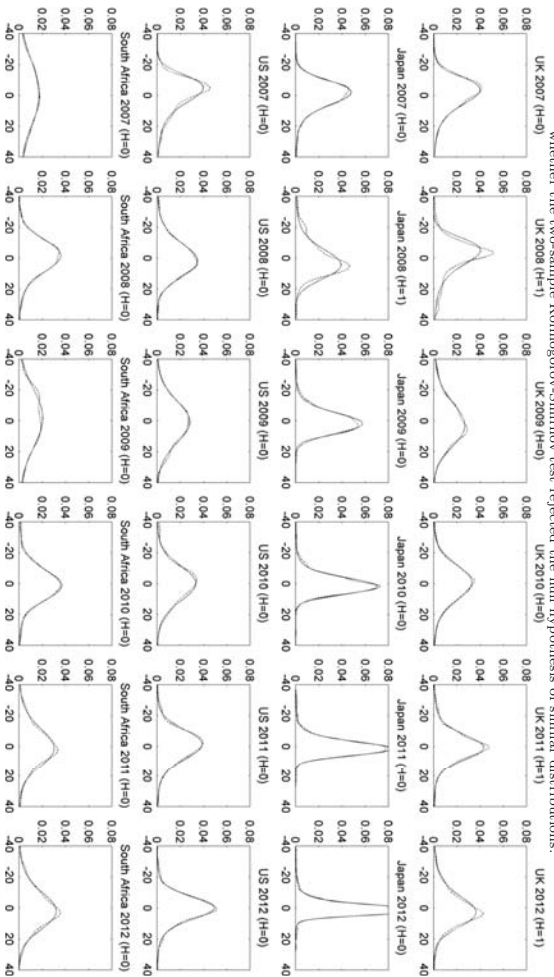


Fig. 1: Empirical and Fitted Stable Distributions

The graphs show the empirical and fitted distributions for the main office markets of four continents for the recent six years, i.e., 2007 to 2012. In the caption it is indicated whether the two-sample Kolmogorov-Smirnov test rejected the null hypothesis of similar distributions.

Fig. 2: Parameter Box Plots Over Time, Office

The graphs show the box plots for the characteristic exponent parameter estimates and how they are dispersed. Here, the results for the office sector for four example countries are shown. The dispersion was calculated by bootstrapping the parameter estimates with 1,000 draws.

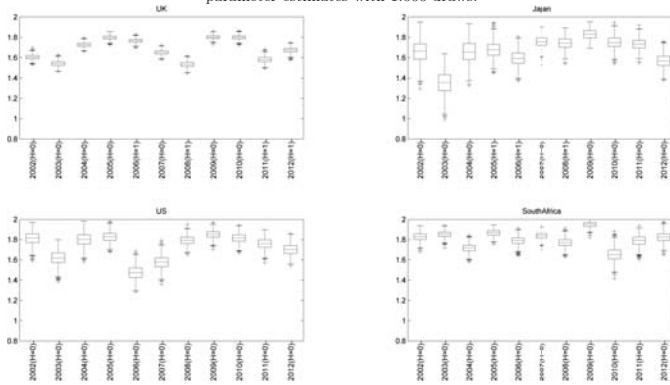


Fig. 3: Parameter Box Plots Over Time, Industrial

The graphs show the box plots for the characteristic exponent parameter estimates and how they are dispersed. Here, the results for the office sector for four example countries are shown. The dispersion was calculated by bootstrapping the parameter estimates with 1,000 draws.

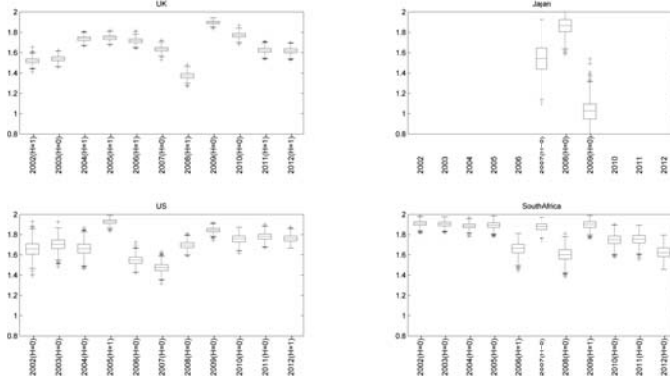


Fig. 4: Comparison of UK and Switzerland Office Markets

The graphs show the empirical and fitted stable distributions for the UK and Switzerland. The captions include the parameter estimates and expected tail loss for comparative reasons.

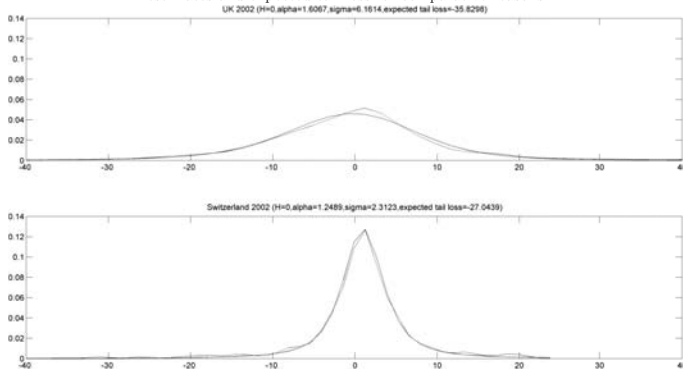
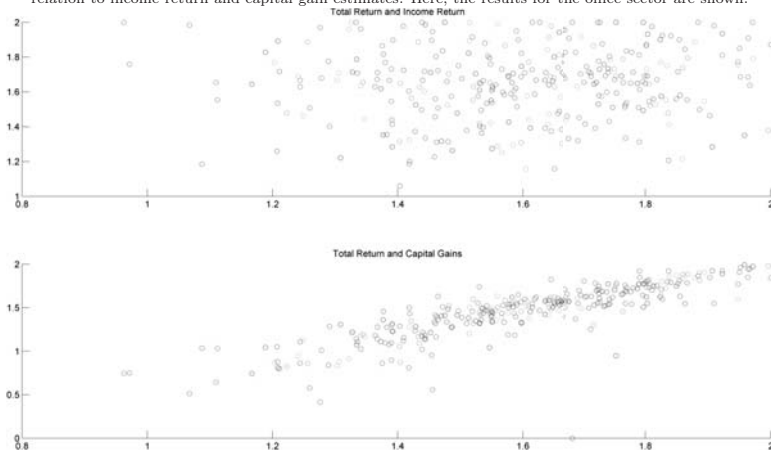


Fig. 5: Relation of Estimates for the Characteristic Exponent

The graph shows all estimates of characteristic exponent parameter estimates where the total return estimates are put in relation to income return and capital gain estimates. Here, the results for the office sector are shown.



Tab. 1: Estimation Results for the Total Returns of the Industry Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	
Austria	α	σ	H	α	σ	H	α	σ	H	α	σ	H
Belgium												
Denmark	1.47	6.39	0	1.2	4.42	0	1.22	4.32	0	1.2	2.83	0
Finland	1.35	5.48	0	1.26	5.3	0	1.57	6.47	1	1.73	8.06	0
France												
Germany												
Italy												
Netherlands	1.3	3.67	0	1.46	4.62	0	1.13	3.01	0	1.37	4.14	0
Norway												
Portugal												
Ireland												
Spain												
Sweden	1.58	10	0	1.69	9.31	0						
Switzerland												
UK	1.52	4.2	1	1.54	4.49	0	1.74	7.46	1	1.75	7.66	1
Australia	1.63	4.64	0	1.82	6.56	0	1.72	4.43	0	1.74	7.29	1
Japan												
New Zealand												
South Korea												
Canada	1.48	3.45	0	1.52	4.79	0	1.78	6.36	0	1.99	11.91	1
USA	1.65	6.34	0	1.7	5.49	0	1.66	6.54	0	1.93	12.07	1
South Africa	1.91	20.86	0	1.9	17.25	0	1.89	16.15	0	1.89	16.57	0

Tab. 2: Estimation Results for the Total Returns of the Office Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012									
	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H						
Austria	1.55	3.34	0	1.45	3.46	0	1.34	3.94	0	1.4	3.29	0	1.49	4.69	0	1.24	3.11	0	1.34	2.53	0	1.51	2.9	0	1.71	2.5	0			
Belgium	1.45	4.05	0	1.24	3.81	0	1.38	3.69	0	1.43	4.37	0	1.39	3.94	0	1.6	3.76	0	1.4	3.31	0	1.49	3.94	0	1.42	5.34	0			
Denmark	1.53	6.74	0	1.46	6.21	0	1.58	7.17	0	1.42	6.52	0	1.6	5.82	0	1.67	5.92	0	1.43	4.84	0	1.47	5.94	0	1.39	6.03	0			
Finland	1.2	3.62	0	1.21	4.28	0	1.17	3.93	0	1.31	3.89	0	1.39	6.23	1	1.34	6.24	0	1.59	5.85	0	1.58	4.71	1	1.30	4.95	0			
France	1.44	4.24	1	1.41	4.77	1	1.6	5.54	1	1.78	7.47	1	1.64	5.07	0	1.61	7.31	1	1.54	5.56	1	1.69	6.17	0	1.5	5.39	0	1.52	4.17	0
Germany	1.3	3.25	0	1.21	3.35	0	1.37	4.16	1	1.36	5.13	0	1.37	5.46	0	1.26	4.03	0	1.44	4.41	0	1.34	3.43	0	1.55	3.37	0	1.43	3.22	1
Italy	1.97	12.74	1	1.38	3.82	0	1.33	4.1	1	0.97	2.42	0	1.41	2.24	0	1.65	3.93	0	1.33	3.28	0	1.31	4.58	0	1.09	1.44	1	1.31	2.13	1
Netherlands	1.57	4.15	1	1.46	3.65	0	1.61	4.37	1	1.54	4.88	0	1.67	5.7	0	1.75	5.51	1	1.68	5.22	0	1.55	4.43	0	1.53	3.76	0	1.58	5.74	0
Norway	1.64	8.62	0	1.55	8.95	0	1.64	8.88	0	1.66	10.68	0	1.53	8.77	0	1.66	11.56	0	1.96	9.87	0	1.61	7.76	0	1.54	4.93	0	1.8	5.07	0
Portugal	1.33	5.53	0	1.63	6.39	0	1.75	4.37	0	1.44	4.26	0	1.28	2.84	1	1.72	4.17	0	1.46	4.02	0	1.51	4.7	1	1.68	2.9	0	1.44	3.65	0
Ireland	1.89	4.11	0	1.68	5.27	0	1.39	3.06	0	1.88	10.19	0	1.9	9.6	0	1.53	3.91	0	1.39	5.29	0	1.56	6.44	0	1.06	5.03	0	1.46	3.83	0
Spain	1.52	6.44	0	1.72	6.27	0	1.48	5.46	0	1.55	6.69	0	1.5	5.71	0	1.48	5.58	0	1.97	9.25	0	1.97	9.25	0	1.66	4.23	0	1.73	5.01	0
Sweden	1.66	6.88	0	1.71	7.34	0	1.67	5.85	0	1.73	6.86	0	1.67	9.7	0	1.64	7.33	0	1.67	9.17	0	1.64	5.79	0	1.62	4.28	0	1.72	3.92	0
Switzerland	1.25	2.81	0	1.21	2.44	0	1.5	2.74	0	1.46	2.74	0	1.51	2.99	0	1.59	3.29	0	1.29	2.6	0	1.35	2.12	0	1.38	3.4	1	1.48	2.99	0
UK	1.61	6.16	0	1.54	6.08	0	1.73	7.57	0	1.8	8.77	0	1.76	9.51	1	1.65	7	0	1.53	6.91	1	1.8	10.47	0	1.8	8.57	0	1.58	6.58	1
Australia	1.7	4.45	0	1.81	6.49	0	1.8	4.17	0	1.91	7.91	0	1.6	6.83	0	1.81	8.68	0	1.69	5.24	0	1.89	7.87	0	1.75	4.52	0	1.58	4.16	0
Japan	1.66	4.53	0	1.37	3.43	0	1.65	3.98	0	1.67	6.27	1	1.59	4.58	1	1.75	5.86	0	1.74	7.17	1	1.83	4.88	0	1.75	3.99	0	1.73	3.98	0
New Zealand							1.19	5.6	0	1.42	5.3	0	1.95	7.03	0	2	4.85	0	1.55	4.54	0	1.82	8.52	0	1.78	3.62	0	1.33	3.53	0
South Korea																														
Canada	1.56	5.07	1	1.71	7.26	1	1.55	6.43	0	2	13.82	0	1.7	9.31	1	1.73	10.14	0	1.66	5.52	0	1.84		0	1.7	1.7	0	1.73	1.73	0
USA	1.82	6.25	0	1.62	5.04	0	1.8	6.84	0	1.83	12.15	0	1.47	5.29	0	1.57	7.01	0	1.79	8.02	0	1.81	1.81	0	1.81	1.81	0	1.76	1.76	0
South Africa	1.83	20.23	0	1.86	15.99	0	1.72	17.96	0	1.87	18.13	0	1.79	11.1	0	1.84	16.3	0	1.77	7.85	0	1.95	1	0	1.66	1.66	0	1.79	1.79	0

Tab. 3: Estimation Results for the Total Returns of the Retail Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012															
	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H												
Austria																																				
Belgium																																				
Denmark	1.37	8.95	0	1.62	9	0	1.73	6.87	0	1.33	6.39	0	1.66	5.99	0	1.47	5.93	0	0.98	5.06	0	1.19	3.88	0	1.49	5.89	0	1.37	4.81	0	1.77	11.87	0			
Finland	1.13	4.35	0	1.04	3.23	0	1.21	6.5	0	1.35	4.9	0	1.44	5.15	0	1.07	6.26	0	1.84	6.01	0	1.61	5.13	0												
France	1.23	5.52	0	1.53	5.86	0	1.47	5.48	0	1.82	9.53	0	1.74	9.54	0	1.63	12.87	1	1.58	6.55	1	1.63	5.66	0	1.61	5.57	1	1.47	5.3	1	1.45	4.53	1			
Germany	1.29	3.65	1	1.3	3.49	0	1.3	3.66	0	1.3	4.04	1	1.2	5.1	0	1.14	4.08	0	1.6	4.78	1	1.4	4.29	0	1.82	5.12	1	1.27	2.8	0	1.62	4.25	0			
Italy																																				
Netherlands	1.33	3.03	0	1.28	2.54	0	1.42	3.62	1	1.57	4.33	1	1.63	5.01	0	1.52	4.03	0	1.49	3.99	1	1.58	3.14	0	1.26	1.82	0	1.31	2.71	0	1.55	3.66	0			
Norway																																				
Portugal	1.26	3.6	0	1.41	3.38	0	1.41	3.62	0	1.72	4.75	0	1.65	5.53	0	1.51	3.8	0	1.81	7.05	0	1.72	3.22	0	1.44	4.14	0	1.66	4.44	0	1.56	4.44	0			
Ireland	1.87	9.19	1	1.81	7.52	0	1.42	5.11	0	1.92	9.76	0	1.85	8.73	0	1.94	5.66	0	1.79	7.72	0	1.89	6.03	0	1.66	4.9	0	1.58	4.57	0						
Spain	1.76	4.95	1	1.7	5.09	0	1.25	3.88	0	1.63	5.39	0	1.58	5.1	0	1.64	6.84	1	1.59	6.11	0	1.54	3.93	0	1.46	3.81	0	1.43	5	1						
Sweden	1.52	6.79	0	1.61	5.66	0	1.41	4.93	0	1.97	7.32	0	1.8	9.25	1	1.58	6.03	0	1.51	5.24	0	1.54	4.78	0	1.37	3.83	0	1.48	4.38	0	1.49	3.49	0			
Switzerland	1	2.64	0	1.32	3.26	0	1.4	2.84	0	1.24	2.34	0	1.35	2.42	0	1.34	3.46	0	1.21	2.69	0	1.18	1.77	0	1.33	2.14	0	1.46	3.1	0	1.46	2.83	0			
UK	1.69	5.58	0	1.61	5.31	0	1.78	7.75	1	1.81	7.55	0	1.71	6.15	0	1.7	5.78	0	1.47	6.36	1	1.91	12.67	0	1.75	6.89	0	1.49	4.4	1	1.58	5.46	0			
Australia	1.49	4.01	0	2	10.7	1	1.61	5.14	0	1.73	7.78	0	1.84	6.2	0	1.83	5.92	0	1.74	5.3	0	1.81	7.5	0	1.66	3.81	0	1.53	3.66	0	1.47	3.08	0			
Japan																																				
New Zealand																																				
South Korea																																				
Canada	1.57	5.59	0	1.25	4.48	0	1.72	8.08	0	1.92	11.88	0	1.47	5.75	0	1.63	7.38	0	1.64	5.13	1	1.45	0	1.79	1.79	0	1.72	1.72	1	1.67	1.67	0				
USA	1.08	3.72	0	1.68	5.41	0	1.69	7.86	0	1.93	11.53	0	1.45	3.53	0	1.55	3.89	0	1.66	6.42	0	1.83	0	1.67	1.67	1	1.85	1.85	0	1.55	1.55	0				
South Africa	1.91	16.55	1	1.78	12.72	1	1.89	15.25	0	1.74	14.14	0	1.67	8.63	0	1.82	11.47	0	1.65	6.78	0	1.99	1	1	1.82	1.82	0	1.76	1.76	0	1.7	1.7	0			

Tab. 6: Estimation Results for the Income Returns of the Industry Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012												
	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H									
Austria																																	
Belgium																																	
Denmark	1.58	2.16	0	1.46	2.19	0	1.65	2.19	0	1.35	1.6	0	1.52	1.64	0	1.86	2.06	0	1.76	1.99	1	1.82	1.82	0				1.67	1.84	0			
Finland	1.47	2.97	1	1.44	2.63	0	1.79	3.41	1	1.49	2.43	1	1.84	3.14	1	1.63	2.28	0	1.53	2.18	0	1.21	1.54	1	1.38	1.61	1	1.47	1.78	1	1.56	2.02	1
France																																	
Germany																																	
Italy																																	
Netherlands	1.14	0.9	0	1.05	1.02	0	1.29	1.22	0	1.61	1.98	0	1.54	1.49	0	1.35	1.08	0	1.33	1.3	0	1.16	1.22	0	1.7	2	0	1.85	2.52	1	1.91	2.5	0
Norway																																	
Portugal																																	
Ireland																																	
Spain																																	
Sweden	1.81	3.74	1	1.59	3.04	0																											
Switzerland																																	
UK	1.49	1.65	1	1.58	1.67	1	1.77	2.01	1	1.59	1.57	1	1.39	1.18	1	1.49	1.18	1	1.66	1.77	1	1.69	2.63	1	1.72	2.16	1	1.65	2.19	1	1.63	2.25	1
Australia	1.89	2.43	1	2	2.51	1	2	2.78	0	1.8	2.23	1	1.5	1.31	1	1.49	1.23	1	1.24	0.97	0	1.83	1.9	1	1.65	1.62	1	1.34	1.06	0	1.21	1.04	0
Japan																																	
New Zealand																																	
South Korea																																	
Canada	1.33	1.61	0	1.34	1.64	1	1.29	1.34	1	1.68	1.87	1	1.34	1.28	1	1.24	1.01	1	1.19	0.85	1	1.07	0	0	0.93	0.93	1	1.4	1.4	0	1.33	1.33	1
USA	1.98	2.95	1	1.93	2.9	1	1.93	2.99	1	1.92	2.36	1	1.89	2.34	1	1.83	2.1	1	1.77	1.98	1	1.5	1	1	1.4	1.4	1	1.62	1.62	1	1.67	1.67	1
South Africa	1.53	4.3	0	1.63	4.58	1	1.63	4.24	1	1.62	2.89	1	1.87	3.18	0	1.42	1.62	1	1.47	1.91	0	1.75	1	1	1.54	1.54	0	1.74	1.74	0	1.33	1.33	0

Tab. 7: Estimation Results for the Income Returns of the Office Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012															
	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H												
Austria				1.79	1.44	0	1.9	1.89	0	1.96	1.67	0	1.65	1.68	0	1.58	1.7	0	1.69	1.56	0	1.71	1.29	0	1.82	1.27	0	1.71	1.27	0						
Belgium				1.71	2.42	1	1.63	1.75	0	1.77	2.03	0	1.89	2.21	0	1.9	2.18	0	1.16	1.15	0	1.06	1.36	0	1.26	1.3	1	1.19	1.36	0						
Denmark	1.29	1.42	0	1.4	1.68	0	1.61	2.11	0	1.8	2.53	0	1.69	1.38	0	1.69	1.38	0	1.44	1.06	0	1.37	1.15	0	1.32	1.3	0	1.44	1.6	0						
Finland	1.77	1.8	0	1.72	1.87	0	1.64	1.88	0	1.65	2	0	1.73	2.2	0	1.66	2.01	0	1.57	1.65	0	1.69	1.73	0	1.67	2.03	0	1.67	2.03	0						
France	1.6	2.29	1	1.72	2.37	1	1.75	2.56	1	1.65	2.53	1	1.71	2.23	1	1.77	2.21	1	1.81	2.24	1	1.63	2.15	1	1.68	2.17	1	1.7	2.24	1	1.66	2.32	1			
Germany	1.67	1.62	1	1.54	1.39	0	1.62	1.7	1	1.63	1.85	0	1.67	2.07	1	1.89	2.21	0	1.72	1.87	1	1.63	1.53	0	1.53	1.44	0	1.5	1.48	0	1.57	1.49	0			
Italy	1.79	1.82	0	1.82	1.67	0	1.79	1.88	0	1.76	1.64	0	1.98	1.75	1	1.98	1.82	1	2	1.97	1	1.71	1.52	1	1.18	0.82	0	1.22	1.13	0	1.52	1.49	1			
Netherlands	1.25	1	0	1.31	1.12	1	1.39	1.47	1	1.66	2	1	1.81	2.15	1	1.73	1.83	1	1.46	1.41	0	1.42	1.53	0	1.49	1.63	0	1.71	2.36	0	1.67	2.68	0			
Norway	1.41	2.08	0	1.61	2.11	0	1.64	2.57	1	1.74	2.39	0	1.58	2.03	1	1.3	1.46	0	1.68	1.74	1	1.44	1.17	0	1.4	1.08	0	1.49	1.13	0	1.6	1.27	0			
Portugal	2	2.81	1	2	2.71	0	2	2.61	0	2	2.51	0	1.97	2.09	0	1.86	2.02	0	1.95	2.29	1	1.89	2.20	1	2	2.42	1	1.99	2.42	1	1.91	2.46	1			
Ireland	1.5	0.84	0	1.33	0.8	0	1.28	0.87	1	2	1.8	1	1.46	0.77	0	1.42	0.73	0	1.41	0.74	1	1.99	2.18	0				1.35	1.97	0	1.51	2.73	0			
Spain	2	2.07	0	2	1.96	0	2	2.46	0	1.76	2.08	1	1.75	1.71	1	1.7	1.68	0	2	1.91	0				1.94	1.92	1	1.78	2	1						
Sweden	1.57	1.76	0	1.69	1.96	0	1.69	1.99	0	1.88	2.13	0	1.77	2.15	0	1.69	1.51	0	1.83	1.71	0	1.47	1.23	0	1.67	1.11	0	1.65	1.03	0	1.72	1.09	0			
Switzerland	1.46	0.72	0	1.26	0.82	0	1.48	1	0	1.56	1.1	1	1.38	0.98	0	1.61	1.1	1	1.4	0.82	0	1.44	0.89	0	1.36	0.76	0	1.35	0.74	0	1.31	0.7	0			
UK	1.58	1.95	1	1.53	2.11	1	1.73	2.47	1	1.66	2.13	1	1.65	1.86	1	1.63	1.67	1	1.65	2.11	1	1.73	3.04	1	1.69	2.77	0	1.65	2.73	0	1.68	3.05	0			
Australia	1.92	2.33	0	1.98	2.64	0	2	2.62	1	1.98	2.26	0	1.71	1.72	0	1.52	1.29	0	1.45	1.16	0	1.55	1.6	0	1.51	1.52	0	1.43	1.67	0	1.38	1.46	0			
Japan	1.81	1.53	1	1.71	1.78	0	1.66	1.56	0	1.95	1.99	0	1.9	1.64	1	1.69	1.25	1	1.6	1.07	0	1.57	1.13	0	1.69	0.99	0	1.76	1.25	0	1.73	1.04	0			
New Zealand				1.83	1.96	0	1.2	0.95	0				1.77	1.82	1	1.38	0.96	0	1.54	1.23	0				1.77	1.77	0	1.71	1.39	0	1.61	1.09	0			
South Korea																																		1.98	1.98	0
Canada	1.64	2.28	0	1.72	2.39	0	1.55	2.05	0	1.87	2.56	1	1.66	1.8	0	1.57	1.49	0	1.48	1.16	1	1.41	0	1.42	1.42	0	1.53	1.33	0	1.52	1.57	0	1.62	1.72	0	
USA	1.53	1.8	0	2	2.86	0	1.87	2.64	0	1.93	2.32	1	1.84	2.2	0	1.87	1.96	0	1.61	1.66	0	1.56	1	1.66	1.56	0	1.69	1.69	0	1.69	1.69	0	1.7	1.7	0	
South Africa	1.55	5.46	0	1.58	5	0	1.64	5.09	0	1.55	4.77	0	1.51	3.95	1	1.48	3.12	1	1.34	2.53	1	1.75	1	1.49	1.49	0	1.65	1.65	0	1.58	1.58	0				

Tab. 8: Estimation Results for the Income Returns of the Retail Properties Sector

Notes: α and σ are the estimates for the characteristic exponent and the scale parameter of a stable distribution. H takes on a value of 1 when a two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the fitted and empirical cumulative density function for the respective country and year are of the same distribution and 0 otherwise.

Country	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012													
	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H	α	σ	H										
Austria							1.68	2.96	0	1.51	2.4	1	1.57	2.19	0	1.52	2.19	1	1.87	2.2	1	1.81	2.3	0	1.99	2.63	0	1.75	2.72	0	1.71	2.87	1	
Belgium																																		
Denmark	1.64	1.7	0	1.49	2.13	0	1.47	1.93	0	1.52	1.92	0	1.55	1.49	0	1.47	1.58	0	1.58	1.97	0	1.25	1.31	0	1.51	1.64	0	1.17	1.32	0	1.23	1.95	0	
Finland	1.47	2.11	0	1.46	2.02	0	1.41	1.97	0	1.52	2.71	0	1.38	2.15	0	1.38	1.47	0	1.49	1.54	0	1.34	1.44	0										
France	1.31	2.4	0	1.62	2.64	1	1.41	1.8	0	1.59	2.07	0	1.56	1.54	0	1.9	2.37	1	1.38	1.1	0	1.4	1.24	1	1.59	1.13	0	1.35	1.23	1	1.39	1.17	0	
Germany	1.69	1.66	0	1.71	1.61	0	1.59	1.8	0	1.71	1.89	0	1.62	1.85	0	1.75	1.94	0	1.58	1.29	1	1.6	1.49	0	1.85	2.05	1	1.64	1.15	0	1.76	1.52	1	
Italy																																		
Netherlands	1.23	0.74	0	1.35	0.76	0	1.34	0.77	0	1.38	0.8	0	1.5	0.78	0	1.53	0.69	0	1.26	0.57	1	1.51	0.73	0	1.63	0.74	0	1.51	0.75	0	1.52	0.9	0	
Norway																																		
Portugal																																		
Ireland	1.92	0.99	0	1.99	0.97	0	2	0.95	0	2	1	0	1.77	0.68	1	1.93	0.61	0	1.65	0.57	0	1.97	1.55	0	1.32	0.95	0	1.41	1.42	0	1.35	1.5	0	
Spain	1.92	2.5	1	1.63	0.51	0	1.18	0.9	0	1.16	0.85	1	1	0.63	1	1.59	1.08	0	1.52	1.08	0	1.47	1.2	0	1.71	1.25	0	1.61	1.46	0	1.5	1.73	0	
Sweden	1.66	1.87	0	1.72	1.97	0	1.76	1.77	0	1.93	1.62	0	1.99	1.66	1	1.79	1.2	0	1.64	1.31	0	1.44	1.22	0	1.61	1.35	0	1.62	1.25	0	1.77	1.53	0	
Switzerland	1.15	0.98	0	1.25	0.88	0	1.56	1.17	0	1.34	0.91	0	1.24	0.88	1	1.44	0.95	0	1.3	0.83	0	1.42	0.87	0	1.55	0.77	0	1.68	0.82	0	1.53	0.88	0	
UK	1.68	1.59	1	1.54	1.35	1	1.51	1.19	1	1.37	0.9	1	1.28	0.76	1	1.49	0.86	1	1.65	1.34	1	1.61	1.87	1	1.59	1.52	1	1.59	1.42	1	1.53	1.52	1	
Australia	1.61	1.45	0	2	2.31	1	1.85	1.87	1	2	2.08	1	1.43	0.88	0	1.04	0.66	1	1.42	0.72	0	1.6	1.18	1	1.53	0.75	1	1.46	0.82	0	1.41	0.93	0	
Japan																																		
New Zealand																																		
South Korea																																		
Canada	1.83	2.85	0	1.71	2.3	0	1.65	2.26	1	1.71	2.13	1	1.51	1.86	1	1.35	1.15	0	1.22	1.01	1	1.24	1	1	1.21	1.21	0	1.33	1.33	0	1.61	1.61	1	
USA	1.21	1.18	0	1.39	1.35	0	1.97	2.16	1	1.18	0.91	1	1.86	1.77	1	1.53	1	1	1.11	0.68	1	1.32	1	1	1.61	1.61	0	1.55	1.56	0	1.41	1.41	0	
South Africa	1.66	3.4	0	1.55	3.32	0	1.73	3.62	1	1.68	2.82	0	1.89	2.94	0	1.67	2.07	0	1.62	1.99	0	1.86	1	1	1.59	1.59	0	1.72	1.72	0	1.66	1.66	0	

