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Generating Random Optimising Choices

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Jan Heufer[1]

Generating Random Optimising Choices

Abstract

This paper provides an efficient way to generate a set of random choices on a set of budgets which satisfy the Generalised Axiom of Revealed Preferences (GARP), that is, they are consistent with utility maximisation. The choices are drawn from an approximate uniform distribution on the admissible region on each budget which ensures consistency with GARP, based on a Markovian Monte Carlo algorithm due to Smith (1984). This procedure can be used to extend Bronars' (1987) method as it can be used to approximate the power of tests for conditions for which GARP is a necessary but not sufficient condition (e.g., homotheticity, separability, risk aversion, etc.). For example, it allows to approximate the probability that a set of random choices which happens to satisfy GARP is also consistent with homotheticity. The approach can also be applied to production analysis and nonparametric tests of cost minimisation.

JEL Classification: C14, C63, D11, D12

Keywords: Cost minimisation; GARP; hypothesis testing; Monte Carlo methods; nonparametric tests; test power; random optimal choices; revealed preference; utility maximisation

July 2012

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1 INTRODUCTION

Afriat's (1967) Theorem shows that for a set of consumption choices from competitive budgets there exists a continuous, monotonic, and concave utility function if and only if the choices satisfy cyclic consistency. Varian (1982) introduced a the *Generalised Axiom of Revealed Preference* (Garp) which is easy to test and equivalent to cyclic consistency.

It is important to know how meaningful the test for utility maximisation is. Therefore, it is helpful to know the probability that random choices violate Garp. This probability can be interpreted as the *power* of the test for utility maximisation against the alternative hypothesis of random choices. Bronars (1987) suggested a Monte Carlo approach to approximate the power: Generate many sets of random choices and tests all of them for Garp. The percentage of sets which violators is the approximate power.

Suppose that the researcher is not only interested in testing Garp but also in testing additional assumption for which Garp is a necessary but not sufficient condition. Such assumptions include homotheticity and different forms of separability (cf. Varian 1983), risk aversion (cf. Heufer 2011), and others. If for example a researcher is interesting in testing a set of observations for consistency with homotheticity, using Varian's (1983) *Homothetic Axiom of Revealed Preference* (Harp), then to approximate the power she can use a variant of Bronars' Power by testing random choices for consistency with Harp. But Garp is a necessary condition for Harp, and Bronars' Power for the Garp test might already be close to unity. That means that very few random choice sets satisfy Garp, and therefore cannot satisfy Harp. It would be useful to know the *conditional* probability that a set of random choices violates Harp given that it satisfies Garp.

A simple way to approximate this conditional probability is to use a rejection technique: Draw a set of random choices and reject it if it does not satisfies Garp. However, Bronars' power can be quite literally 100%, that is, even out of thousands of sets of random choices no set satisfies Garp. This is the case in the fifty budget experiments conducted by Fisman et al. (2007) and Choi et al. (2007).

The contribution of this paper is to introduce a Monte Carlo approach to generate sets of random choices which satisfy Garp, using a Markovian method introduced by Smith (1984). He showed that a symmetric mixing algorithm, which generates a Markov chain on a bounded region, will generate a sequence of points asymptotically uniformly distributed within the region. This algorithm is used to generate sets of random choices on budgets which are uniformly distributed such that the sets satisfy Garp. These Garp-consistent random sets can then be tested for consistency with other conditions such as Harp. The algorithms presented here can also be applied to nonparametric approaches to production analysis (cf. Varian 1984).

Garp has been tested in numerous experimental papers, such as Sippel 1997, Mattei 2000, Harbaugh and Krause (2000), Harbaugh et al. (2001), Andreoni and Miller (2002), Chen et al. (2006) Fisman et al. (2007), and Choi et al. (2007). For papers who test for Garp, computing Bronars' Power has become a standard. Some authors (Sippel 1996, Heufer 2012a) have pointed out that if one allows real consumers to deviate from perfect utility maximisation and computes efficiency indices such as Afriat's Efficiency Index (Aei), then the power of the test can be reduced substantially. Heufer (2012a), in particular, provides a procedure to compute the loss of power depending on the Aei and suggests a method to determine the optimal tradeoff between power and the number of consumers accepted as close enough to Garp. The approach presented here could be extended by first generating sets of Garp-consistent choices and then computing efficiency indices for the additional condition.

Section 2 introduces the notation and basic concepts used in the paper. Section 3 introduces the algorithms which lead to the suggested method. Section 4 concludes.

2.1 *General Definitions and Concepts*

A set of observed consumption choices consists of a set of chosen bundles of commodities and the prices and incomes at which these bundles were chosen. Let $\mathbb{R}^L_+$ be the commodity space, where $L \geq 2$ denotes the number of different commodities.[1] The price space is $\mathbb{R}^L_{++}$. Consumers choose bundles $x^i = (x^i_1, \ldots, x^i_L)' \in \mathbb{R}^L_+$ when facing a price vector $p^i = (p^i_1, \ldots, p^i_\ell) \in \mathbb{R}^L_{++}$; a budget is then defined by $B^i = B(p^i) = \{x \in \mathbb{R}^L_+ : p^i x^i \leq 1\}$. That is, prices are normalised such that expenditure always equals 1; we will therefore also identify budgets with their characteristic price vector. The entire set of N observations on a consumer is denoted as $\Omega = \{(x^i, p^i)\}_{i=1}^N$.[2] Let $\bar{B}^i = \bar{B}(p^i) = \{x \in \mathbb{R}^L_+ : p^i x^i = 1\}$.

An observation x^i is *directly revealed preferred* to x, written $x^i \, \mathrm{R}^0 \, x$, if $p^i x^i \geq p^i x$; it is *revealed preferred* to x if $x^i \, \mathrm{R} \, x$, where R is the transitive closure of R^0; it is *strictly directly revealed preferred* to x, written $x^i \, \mathrm{P}^0 \, x$, if $p^i x^i > p^i x$. A budget $B(p)$ is *directly revealed preferred* to a budget $B(p^i)$, written $p \, \mathrm{R}^0_B \, p^i$ if $px^i \leq 1$, where x^i is the observed choice on $B(p^i)$; it is *revealed preferred* to $B(p^i)$ if $p \, \mathrm{R}_B \, p^i$, where R_B is the transitive closure of R^0_B; it is *strictly directly revealed preferred* to $B(p^i)$ if $p \, \mathrm{P}^0_B \, p^i$ if $px^i < 1$; is is *strictly revealed preferred* to $B(p^i)$, written $p \, \mathrm{P}_B \, p^i$ if there exist observed budgets $B(p^j)$ and $B(p^k)$ such that $p \, \mathrm{R}_B \, p^j$, $p^j \, \mathrm{P}_B \, p^k$, and $p^k \, \mathrm{R}_B \, p^i$.

A utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ *rationalises* Ω if $u(x^i) \geq u(x)$ whenever $x^i \, \mathrm{R} \, x$. We say that Ω satisfies the *Generalised Axiom of Revealed Preference* (GARP) if $x^i \, \mathrm{R} \, x^j$ implies $\left[\text{not } x^j \, \mathrm{P}^0 \, x^i\right]$. It can then be shown (Afriat 1967, Diewert 1973, Varian 1982) that there exists a continuous, monotonic, and concave utility function that rationalises Ω if and only if Ω satisfies GARP.

2.2 *Supporting Bundles and Forcasting Choices*

Following Varian (1982), we define the set of bundles which *support* a price vector p^0 not previously observed as

$$S(p^0 | \{(x^i, p^i)\}_{i=1}^N) = \{x^0 \in \mathbb{R}^L_+ : \{(x^i, p^i)\}_{i=0}^N \text{ satisfies GARP and } p^0 x^0 = 1\}. \tag{1}$$

That is, $S(p^0|\Omega)$ is the set of all bundles which can be chosen on $B(p^0)$ without violating GARP when combined with the previous observations. Any utility maximising consumer for whom we have observed Ω will choose a bundle in $S(p^0)$ when facing the new budget $B(p^0)$. See Figure 1 for an illustration. The set $S(p^0)$ is described by a linear system, as Fact 1 shows.

Fact 1 (Varian 1982) *A bundle x^0 is in $S(p^0|\Omega)$ if and only if it satisfies the conditions*
(1) $p^0 x^0 = 1$,
(2) $p^i x^i \leq p^i x^0$ *for all* p^i *such that* $p^0 \, \mathrm{R}_B \, p^i$,
(3) $p^i x^i < p^i x^0$ *for all* p^i *such that* $p^0 \, \mathrm{P}_B \, p^i$.

We also need an open subset $T(p^0|\Omega)$ of $S(p^0|\Omega)$, defined as the set of all $x^0 \in \mathbb{R}^L_{++}$ such that
(1) $x^0 \in S(p^0|\Omega)$,

[1] The following notation is used: For all $x, y \in \mathbb{R}^L$ we write $x \geq y$ for $x_i \geq y_i$ for all i, $x > y$ for $x_i \geq y_i$ and $x \neq y$ for all i, and $x \gg y$ for $x_i > y_i$ for all i. We denote $\mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x \geq 0\}$ and $\mathbb{R}^L_{++} = \{x \in \mathbb{R}^L : x \gg 0\}$.

[2] Strictly speaking, we observe budgets as a pair $(q^i, w^i) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}$ and then set $p^i = q^i / w^i$. The implicit assumption here is that demand is homogeneous.

(2) $p^i x^i < p^i x^0$ for all p^i such that $p^0 \mathrm{R}_B\, p^i$ and $p^0 \neq p^i$.
We need the set T for technical reasons explained in the next section. It is straightforward to show that the set $T(p^0|\Omega)$ is the relative interior of $S(p^0|\Omega)$; that is, the interior of $S(p^0|\Omega)$ within the subspace defined by the budget hyperplane $\bar{B}(p^0)$.

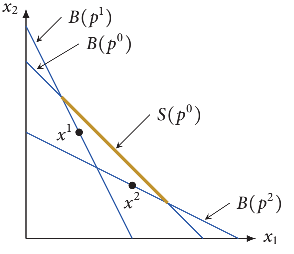


Figure 1: The set $S(p^0)$.

2.3 *Power against Random Behaviour*

Depending on the set of budgets, the probability that a set of random choices violates Garp can differ substantially. It is rarely feasible to compute the exact probability, which is why Bronars (1987) suggested a Monte Carlo approach to determine the power the test has against random behaviour. The approximate power (Bronars' Power) of the test is the percentage of random choice sets which violated Garp. Bronars' first algorithm, which we will focus on here, follows Becker's (1962) example by inducing a uniform distribution on the budget hyperplane.

3 ALGORITHMS

3.1 *Preliminary Algorithms*

Algorithm 1, the *simplex point picking algorithm*, returns a random point uniformly distributed on the unit simplex (see for example Tempo et al. 2005, p. 245).

Algorithm 1
Input: An integer $L \geq 2$.
Output: A random point X *uniformly distributed on the* $L-1$ *unit simplex.*

1. *Generate* L *independent random variables* $Y = (Y_1, \ldots, Y_L)$ *from the Gamma distribution with parameters* $\alpha = \beta = 1$.
2. *Set* $X = Y/(\sum_{i=1}^{L} Y_i)$ *and return* X.

The simplex point picking algorithm can then be used to generate a random choice on a budget from a uniform distribution. Algorithm 2 does this for each budget in a set of N budgets; it can be used to compute Bronars' power.

Algorithm 2

Input: A set of N normalised price vectors $\{p^i\}_{i=1}^N$.

Output: A set of random choices on each $B(p^i)$ *uniformly distributed on* $\bar{B}(p^i)$.

1. *Set* $k = 1$.
2. *Generate a point X on the* $L-1$ *simplex using Algorithm 1. Set* $x^k = (X_1/p_1^k, \dots, X_L/p_L^k)$. *Set* $k = k+1$.
3. *If* $k = N$, *stop and return* $\{x^i\}_{i=1}^N$. *Otherwise, go to Step 2.*

For any set $\{p^i\}_{i=1}^N$, we can execute Algorithm 2 many times and test all the generated choice sets for Garp. Bronars' power is then the percentage of choice sets which do not satisfy Garp.

Let $D = \{d \in \mathbb{R}^L : \|d\| = 1\}$ be the unit sphere. Selection a random element $d \in D$ from a uniform distribution on D is equivalent to selecting a random direction in $\mathbb{R}^L$. Algorithm 3, the *random direction algorithm*, generates such a random direction (see for example Knuth 1998 [1969], p. 135).

Algorithm 3

Input: An integer $L \geq 2$.

Output: A random point $d \in D$ *uniformly distributed on* D.

1. *Generate L independent normally distributed random variables,* $\delta = (\delta_1, \dots, \delta_L)$.
2. *Set* $d = \delta / \|\delta\|$ *and return* d.

Algorithm 4, the *mixing algorithm*, can be found in Smith (1984).

Algorithm 4

Input: An integer $M \geq 1$ *and a set* $\Theta \subseteq \mathbb{R}^L$ *with* $L \geq 2$.

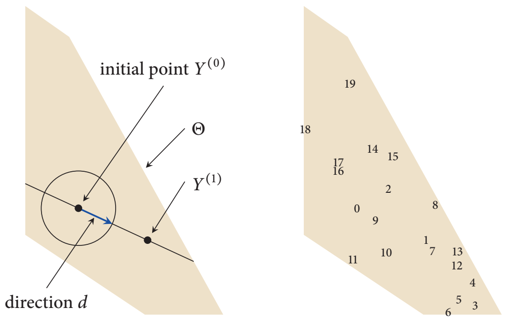
Output: A Markov chain of points $Y^{(0)}, \dots, Y^{(M)}$ *in* Θ.

1. *Set* $k = 0$. *Choose an initial point* $Y^{(0)} \in \Theta$.
2. *Generate a random direction* $d \in \mathbb{R}^L$ *using Algorithm 3.*
3. *Set* $\mathcal{L} = \Theta \cap \{x \in \mathbb{R}^L : x = Y^{(k)} + \lambda d\}$, *where* $\lambda \in \mathbb{R}$.
4. *Generate a random point* $Y^{(k+1)}$ *uniformly distributed on* $\mathcal{L}$.
5. *If* $k = M$, *stop and return* $Y^{(0)}, \dots, Y^{(M)}$. *Otherwise, set* $k = k+1$ *and go to Step 2.*

Algorithm 4 generates a Markov chain of points in Θ. Smith (1984) showed that under certain conditions the generated points approach a uniform distribution over the region for any starting point $Y^{(0)}$. In particular, the assumptions are satisfied if Θ is an open and bounded subset of $\mathbb{R}^L$ with $L \geq 2$ and Θ is itself L-dimensional (i.e., the affine hull of Θ is of dimension L). See Figure 2 for an illustration.

3.2 *The Two-Dimensional Case*

The method based on the mixing algorithm only works for commodity spaces with $L \geq 3$, as in the two-dimensional case the set $S(p^0)$ is only a line segment (i.e., one-dimensional; see Figure 1). As many induced budget experiments only consider two goods, the two-dimensional case is quite relevant, which is why we treat it separately here. Algorithm 5 generates a set of choices, $\{x^i\}_{i=1}^N$, on a set of budgets with price vectors $\{p^i\}_{i=1}^N$, such that $\{(x^i, p^i)\}_{i=1}^N$ satisfies Garp.



(a) The first step of Algorithm 4. (b) Markov chain generated by Algorithm 4.

Figure 2: An illustration of the first step of Algorithm 4 and the first twenty points generated by it.

Algorithm 5

Input: A set of $N \geq 2$ *normalised price vectors* $\{p^i\}_{i=1}^N$ *with* $p^i \in \mathbb{R}^2_{++}$ *for* $i = 1, \ldots, N$.

Output: A set of choices $\{x^i\}_{i=1}^N$ *drawn from a uniform distribution on* $\{\bar{B}(p^i)\}_{i=1}^N$ *such that* $\{(x^i, p^i)\}_{i=1}^N$ *satisfies* GARP.

1. *Set* $k = 1$. *Generate the first choice* x^k *on* $B(p^k)$ *using Algorithm 2.*
2. *Set*

$$\begin{aligned}
\tilde{x}^{\min} &= \arg\min_{x^0 \in S(p^{k+1} | \{(x^i, p^i)\}_{i=1}^k)} x_2^0 \\
\tilde{x}^{\max} &= \arg\max_{x^0 \in S(p^{k+1} | \{(x^i, p^i)\}_{i=1}^k)} x_2^0 \\
\phi &= (\tilde{x}_1^{\max}, \tilde{x}_2^{\min}) \\
\tilde{p} &= ([\tilde{x}^{\min} - \phi_1]^{-1}, [\tilde{x}^{\max} - \phi_2]^{-1})
\end{aligned}$$

3. *Set* $k = k+1$. *Generate a point* X *on the* $L-1$ *simplex using Algorithm 1. Set* $x^k = (X_1/\tilde{p}_1, \ldots, X_L/\tilde{p}_L) + \phi$.
4. *If* $k = N$, *stop and return* $\{x^i\}_{i=1}^N$. *Otherwise, go to Step 2.*

We omit the proof that Algorithm 5 generates a set of GARP-consistent observations, as it is rather straightforward. Step 2 computes ϕ, which is used as a new "origin"; then $\tilde{p}$ describes a new budget which if translated by ϕ equals $S(p^{k+1} | \{(x^i, p^i)\}_{i=1}^k)$. Step 3 generates a random choice on $\tilde{p}$ and then translates it to obtain a choice on $S(p^{k+1} | \{(x^i, p^i)\}_{i=1}^k)$. Most importantly, the algorithm does not simply compute a set of GARP-consistent choices, but does so by drawing random choices from a uniform distribution over the admissible region given by the set of supporting bundles, conditional on the previously generated choices. Algorithm 5 can be executed many times, using random permutations of the budgets such that each budget is equally likely to be the kth one. This will provide many sets of random choices which satisfy GARP and can then be tested for additional assumptions.

3.3 *The Higher-Dimensional Case*

We cannot directly use Algorithm 4 as the set $S(p^0|\Omega)$ for the commodity space $\mathbb{R}^L_+$ is a subset of the budget hyperplane $\bar{B}(p^0)$ and thus only of dimension $L-1$. Therefore we suggest to use Algorithm 6, which is explained below.

Algorithm 6

Input: An integer $M \geq 1$ and set of $N \geq 2$ normalised price vectors $\{p^i\}_{i=1}^N$ with $p^i \in \mathbb{R}^L_{++}$ with $L \geq 3$ for $i = 1, \ldots, N$.

Output: A set of choices $\{x^i\}_{i=1}^N$ drawn from a uniform distribution on $\{\bar{B}(p^i)\}_{i=1}^N$ such that $\{(x^i, p^i)\}_{i=1}^N$ satisfies Garp*.*

1. *Set $k = 1$. Generate the first choice x^k on $B(p^k)$ using Algorithm 2.*
2. *Set $k = k+1$. Set $\mathcal{T} = T(p^k|\{(x^i, p^i)\}_{i=1}^{k-1})$.*
3. *Set $\ell = 0$. Choose an initial point $Y^{(\ell)} \in \mathcal{T}$.*
4. *Set $\ell = \ell + 1$. Generate a random direction $d \in \mathbb{R}^L$ using Algorithm 3. Set*

$$\mathcal{T}' = \{x \in \mathbb{R}^L_{++} : x_j = Y_j^{(\ell)} + \lambda d_j \text{ for } j = 1, \ldots, L-1 \text{ and } (x_1, \ldots, x_L) \in \bar{B}(p^k)\},$$

   where $\lambda \in \mathbb{R}$. Set $\mathcal{L} = \mathcal{T} \cap \mathcal{T}'$.
5. *Generate a random point $Y^{(\ell)}$ uniformly distributed on $\mathcal{L}$.*
6. *If $\ell = M$, set $x^k = Y^{(\ell)}$ and go to Step 7. Otherwise, go to Step 4.*
7. *If $k = N$, stop and return $\{x^i\}_{i=1}^N$. Otherwise go to Step 2.*

The first three steps are straightforward. Note that we use the set T instead of S. The set S is closed (within the budget hyperplane), whereas T is open; thus, using T assures that Smith's (1984) conditions are satisfied. This is not a strong limitation except for the choice of the initial point, as the probability of drawing a point on the boundary is zero.

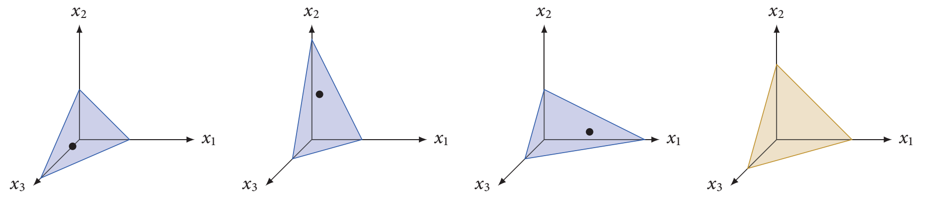
The first time Step 4 is reached, it computes a line $\mathcal{T}$ in the budget hyperplane $\bar{B}(p^2)$ through the initial point $Y^{\{(0)\}}$ using a random direction in $\mathbb{R}^{L-1}$. Given λ and d and the fact that $\mathcal{T}' \subset \bar{B}(p^2)$, the value of x_L is unique. Step 5 then generates a point uniformly distributed on the intersection of the line $\mathcal{T}'$ with the admissible set $\mathcal{T}$ such that the generated points can be used as choices which satisfy Garp. Algorithm 6 therefore generates a Markov chain in the $L-1$ dimensional subspace defined by the budget hyperplanes.

Generating a random point on $\mathcal{L}$ in Step 5 is particularly easy because S and T are convex sets; this follows from Fact 1. Thus, we can simply compute the minimal and maximal λ such that $\mathcal{L} \neq \varnothing$ and then draw λ from a uniform distribution on that interval.

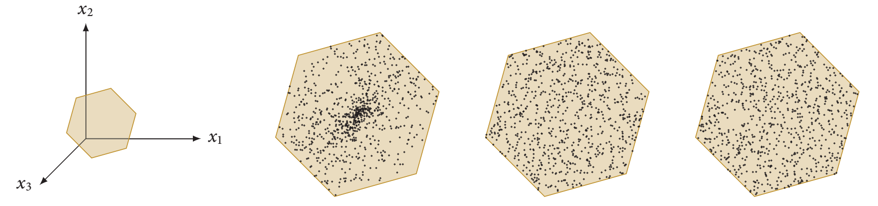
Again, we omit the full proof that the set generated by the algorithm satisfies Garp, as it is straightforward. See Figure 3 for an illustration. Note that the points generated with $M = 15$ already appear to be uniformly distributed on T; there is little difference to the distribution of points generated with $M = 40$.

4 CONCLUSION

This paper provides an efficient procedure to generate random choices from a uniform distribution which satisfy Garp. An immediately useful application of the procedure is an extension of Bronars' (1987) method to approximate the power of a nonparametric test with Monte Carlo methods for conditions for which Garp is a necessary but not sufficient condition. It can be used to approximate the conditional probability



(a) Budget hyperplane $\bar{B}(p^1)$ and choice x^1. (b) Budget hyperplane $\bar{B}(p^2)$ and choice x^2. (c) Budget hyperplane $\bar{B}(p^3)$ and choice x^3. (d) Budget hyperlane $\bar{B}(p^4)$.



(e) The set $S(p^4|\{(x^i, p^i)\}_{i=1}^3$ and its relative interior. (f) 800 points with $M = 2$. (g) 800 points with $M = 15$. (h) 800 points with $M = 40$.

Figure 3: The first three budgets and the choices are shown in (a) - (c). (d) shows the fourth budget. (e) shows the admissible region on the fourth budget given the three previous choices. (f) - (h) show 800 points generated using parts of Algorithm 6, with different M.

that a set of random choices satisfies a condition other than utility maximisation, given that it also satisfies GARP. A possible generalisation is to generate sets of GARP-consistent choices and compute efficiency indices for the additional condition, such as homothetic efficiency (c.f. Heufer 2012b) or stochastic-dominance efficiency (c.f. Heufer 2011). The approach can be further extended by generating choice sets which satisfy other conditions, such as HARP, and then to approximate the conditional probability that it also satisfies another condition, such as separability. Finally, the procedure can also be applied to testing assumptions on cost functions (see, e.g., Varian 1984) when we observe input and output data instead of consumption choices.

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