RUHR

ECONOMIC PAPERS

Jan Heufer

Crime, Inequality, and the Private Provision of Security

technische universität dortmund

#269

Imprint

Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
Hohenzollernstr. 1-3, 45128 Essen, Germany

Editors

Editorial Office

Ruhr Economic Papers #269

Responsible Editor: Wolfgang Leininger

ISSN 1864-4872 (online) – ISBN 978-3-86788-314-6

Ruhr Economic Papers #269

Jan Heufer

Crime, Inequality, and the Private Provision of Security

technische universität
dortmund

Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über: *http://dnb.d-nb.de* abrufbar.

ISSN 1864-4872 (online)
ISBN 978-3-86788-314-6

Jan Heufer[1]

Crime, Inequality, and the Private Provision of Security

Abstract

In a high-crime environment with many high-income citizens, private security companies which offer protection against crime can flourish. In this article crime is modelled as a game where richer victims yield a higher return on crime, but with decreasing returns to crime as more criminals choose crime to supplement their income. Private security providers offer protection against crime and face Cournot competition. The model allows for the analysis of market clearing prices for effort against crime. Among the implications of the model are that rising inequality will lead to more expenditure on protection against crime, and that the upper income classes are suffering from the same or lower crime density than the middle income class. Taking into account the response of criminals and victims, rising inequality can actually lead to less crime if either (i) the legal income opportunity of the marginal criminal increases or (ii) marginal utility from income decreases and richer individuals spend a higher proportion of their income on protection (i.e. protection is a superior good). Often the middle class suffers from higher crime densities as inequality increases, as the increased spending on protection by the upper class (i) shifts crime to the middle class and (ii) increases market prices for protection, leaving the middle class with less affordable protection against crime. Emigration of the middle class can then further increase inequality. This highlights the importance of taking into account the response of individuals against crime and shows that the link between inequality and crime is a complex one.

JEL Classification: D10, D30, D40, K42, L10

Keywords: Crime; inequality; competition of security companies; private enforcement of law; private provision of security

July 2011

---

1 TU Dortmund. – Views expressed represent exclusively the author's own opinions. *Thanks to Burkhard Hehenkamp, Wolfgang Leininger, Tobias Wenzel, Anil Yildizparlak, and the participants of the Spring Meeting of Young Economists 2011 in Groningen, the Royal Economic Society Annual Conference 2011 in London, and the Volkswirtschaftliches Kolloqium at TU Dortmund University. –* *All correspondence to Jan Heufer, TU Dortmund, 44221 Dortmund, Germany, E-Mail: jan.heufer@tu-dortmund.de.*

1 INTRODUCTION

This article focuses on property crime directed against private individuals or households, the response of victims to crime, the effect of the income distribution on the crime level, the distribution of crime among households, and the distribution and effect of private effort against crime.

While private demand and supply for protection against crime is not uncommon, in many countries this protection is mostly of a passive nature, whereas the active fight against crime is the job of the public police. However, in a high-crime environment with an under-supplied police force, high inequality, and many high-income citizens, private security companies which offer the service of a regular police to paying customers can flourish. As of 2009, more than 375,000 private security officers are active in South Africa,[1] working for 6,392 private security providers.[2] These private security officers and the armed response vehicles of private security companies outnumber the members of the public South African Police Service (SAPS) and police vehicles by 3:1 (Goodenough 2007). Of the private security companies, 1,181 are registered as "active armed response businesses" as of 2007. According to surveys, South Africans trust the private security industry far more than they trust the SAPS (Prinsloo and Marais 2006).

The pioneering work in the economic analysis of crime is due to Becker (1968). He treats criminals as rational individuals who decide to become criminal solely based on expected utility. He uses a social welfare function to derive optimal policies that minimize the cost of crime to society. A rather short section of his paper is devoted to private expenditures against crime, and the supply side of protection is not explicitly modelled. While Becker's work inspired many empirical tests of its implications and further theoretical discussions about crime (e.g. Ehrlich 1973, Ehrlich 1975, Block and Heineke 1975, Witte 1980, Ehrlich 1996, Corman and Mocan 2000, Di Tella and Schargrodsky 2004), the analysis of private demand and supply for protection and

[1] The number excludes guards employed in-house.

[2] Data according to the Annual Reports 2006/2007 and 2008/2009 of the South African Private Security Industry Regulatory Authority (PSIRA).

its effects has received less attention. Becker and Stigler (1974), Landes and Posner (1975), and Polinsky (1980) discuss the possibilities of private law enforcement when enforcers receive a fraction of the fine paid by the criminals; see also Friedman (1984). Effects of private precautions against crime have been considered by Clotfelter (1978), Shavell (1991), Ben-Shahar and Harel (1995), and Helsley and Strange (1999).

One argument for a public tax-financed police force is that protection against crime is a human right, and that therefore the supply of security for citizens, independent of their income, is a prime task for any state. An effective executive is also a cornerstone of any functioning state. Besides arguments from ethics and jurisprudence, supplying protection might exhibit increasing returns to scale, leading to a natural monopoly; and one might argue that, given the delicate nature of the business, it is best left to the state. However, as in the case of South Africa, there is evidence that suggests that the security industry can be quite competitive.

Economic theory predicts that higher inequality can lead to more crime, based on opportunity costs and incentives. We might also expect exposure to property crime to be relatively higher among richer households than among poorer households because richer individuals are more attractive targets. The empirical evidence for these predictions is somewhat mixed: For example, Witt et al. (1999) find that inequality increases crime; Kelly (2000) finds that inequality has no effect on property crime, but poverty has a positive effect; Choe (2008) finds a strong and robust effect of relative income inequality on burglary; Neumayer (2005) does not find a strong effect of income inequality on theft. Di Tella et al. (2002) find that for home robberies victims of increased crime levels in Argentina have mostly been poor, whereas both rich and poor have suffered similar increases in victimization for street robberies; Witt et al. (1999) find that availability of “thievable property” increases crime; Bernasco and Nieuwbeerta (2005) find that the effect of neighbourhood affluence as measured by real estate value has no significant effect on crime and that burglars do not show an apparent preference for wealthy neighbourhoods; Demombynes and Berk (2005) show that burglary rates in South Africa are higher in wealthier police jurisdictions and that burglars travel to neighborhoods with higher expected returns; Evans (1989: 93–94) summarizes evidence which suggests that burglars even prefer low-status neighbourhoods.

One possible reason for these mixed results is that (potential) victims of crime respond to crime rates by appropriate precautions against crime, partly to directly protect their property, partly to deter crime in the first place and perhaps to shift crime to other, less protected, individuals. For example, Di Tella et al. (2002) present evidence that rich households are better able to protect their homes through private security; Collett-Schmitt (2007) finds that burglar alarms have a significant and negative effect on burglary rates. Gonzalez-Navarro (2008) finds that while the introduction of the vehicle tracking technology Lojack for certain publicly know car types in some Mexican states reduced theft for Lojack cars by 55%, most of this reduction in theft was offset by increased theft in neighbouring states.

The idea that individual precautions can shift crime from those who invest in private security to those who do not is not new to the theoretical literature (e.g. Clotfelter 1978, Shavell 1991). The case of South Africa suggests that this effect can be real, and can increase social inequality in a heterogeneous society. Goodenough (2007) quotes the manager of the South African municipality of eThekwini:

> Distances, both social and physical, then divide people even further and the more resourced group decides to spend more money on privatized security. The poor in turn become more reliant on a security force whose members are often paid less than that of the private sector.

Similarly, Irish (1999) notes that in South Africa

> [t]he expansion of private security involvement, such as patrolling the neighbourhoods of those who can afford it, has the effect of creating 'enclaves'. There may be a reduction in crime in an enclave, but this does not lead to an overall reduction in crime.

The model of Barenboim and Campante (2008) suggests that the (possible) causality between inequality and crime can also be inverted; in fact they show empirically that partly due to emigration crime can increase inequality.

In this article, crime is modelled as a game with a measure space of players with different legal income. Richer victims yield a higher return on crime, but the return to crime directed against a particular income type decreases as more criminals target that income type. Private security providers offer protection against crime and face Cournot competition. The model allows for the analysis of market clearing prices for effort against crime. Some of the implications of the model are that rising inequality will lead to more expenditure on protection against crime, and that the upper income classes are suffering from the same or lower crime density than the middle income class. Taking into account the response of criminals and victims, rising inequality can actually lead to less crime if either (i) the legal income opportunity of the marginal criminal increases or (ii) marginal utility from income decreases and richer individuals spend a higher proportion of their income on protection (i.e. protection is a superior good). Often the middle class suffers from higher crime densities as inequality increases, as the increased spending on protection by the upper class (i) shifts crime to the middle class and (ii) increases market prices for protection, leaving the middle class with less affordable protection against crime. Emigration of the middle class can then further increase inequality. This highlights the importance of taking into account the response of individuals against crime and shows that the link between inequality and crime is a complex one.

The rest of the paper is organized as follow. Section 2 introduces the basic setup of an economy with a measure space of individuals and the utility functions characterizing them. It is shown that an equilibrium crime distribution exists, that poor individuals supplement their income with crime, while rich individuals do not, that inequality tends to increase crime but this is not a necessity, that higher income of high income types can decrease the harm from crime suffered by poor and intermediate income types and vice versa. Section 3 introduces private security companies which supply the economy with effort against crime. They face Cournot-competition; the model allows in principle to analyse market clearing prices for security. It shows that an equilibrium exists; that richer individuals will demand more effort than poorer individuals; that individuals for whom it is optimal not to demand effort never benefit and often suffer from the private provision of security; that effort against crime is a superior good and an increase in income of rich individuals will increase the market price for effort, leaving poor and intermediate income types worse off than before.

2 THE BASIC SETUP

2.1 *Preliminaries*

We consider a non-atomic economy with a population that consists of a continuum of *individuals* (or players), given by the unit interval $Q = [0,1]$ endowed with Lebesgue measure μ. Individuals have a certain *legal income* $y \in \mathbb{R}_+$, which we will also refer to as the *income type* of an individual. Income is distributed according to a *(cumulative) distribution function* $F : \mathbb{R}_+ \to [0,1]$, i.e $F(y)$ represents the proportion of individuals with income less than or equal to y. A function $Y : Q \to \mathbb{R}_+$ is referred to as the *income distribution*, with $Y_q = F^{-1}(q) = \inf_y\{y : F(y) \geq q\}$, i.e. Y_q gives the income of the individual at $q \in Q$. We will also refer to a $q \in Q$ as the income type, meaning income Y_q, when the meaning is clear. Note that the density at every q is unity. Let $\mathcal{D}$ denote the set of all income distributions.

Individuals have a *strategy set* $A = Q \cup \{-1\}$. A strategy $a \in Q$ is interpreted as becoming a criminal and stealing from individuals of income type a to supplement the legal income. The strategy $a = -1$ is interpreted as not becoming a criminal. For a clearer exposition, we will denote $A_C = Q$. Let $\mathcal{M}$ be the set of Borel probability measures on A.

Each individual is characterized by a *utility function* $U : A \times \mathcal{M} \to \mathbb{R}$; let $\mathcal{U}$ be the set of all utility functions. Given the strategy $a_q \in A$ chosen by the individual q and the strategy distribution ν of all individuals, $U_q(a_q, \nu)$

represents the utility of that individual. A *game of crime* is then characterized by a Borel measure μ on $\mathcal{U}$.

Let χ be a Borel measure on $\mathcal{U} \times A$, and let $\chi_{\mathcal{U}}$ and χ_A denote the marginals of χ on $\mathcal{U}$ and A, respectively. We interpret $\chi_A(a) = \int_{q \in Q} \chi(U_q, a)\,\mathrm{d}\,q$, $a \in A_C$, as the *crime density* at income type a, and $c^* = 1 - \chi_A(-1)$ as the *crime rate*.

2.2 The Utility Functions

Before we specify the utility functions of the individuals, we adopt a simplifying informal assumption to keep the model tractable:

Assumption 1. *Only legal income Y_q can be stolen.*

Remark 1. *Technically, we need Assumption 1 to avoid recursive definitions. Without the assumption, the strategy sets would have to be expanded to allow to choose not only to steal from individuals of income type q, but also to choose to steal from individuals of income type q who choose to steal from individuals of income type q' who choose to steal from individuals of income type q'' etc.*

The income from stealing will be referred to as the *crime payoff* or simply payoff to avoid confusion with the (legal) income. The payoff from stealing of individuals of income type q depends on their income Y_q and the crime density at q. This payoff can be represented by a continuous integrable function $S : Q \times \mathbb{R}_+ \to \mathbb{R}_+$. We use the shorthand notation S_a for the payoff of strategy $a \in A_C$ whenever the meaning is clear.

Individuals of type q will experience *disutility from criminal activity* directed against them, i.e. from individuals who choose strategy $a = q$. This disutility depends on the crime density at q and can be represented by a continuous function $H : Q \to \mathbb{R}_+$. We use the shorthand notation H_q whenever the meaning is clear.

We assume that individuals who choose a strategy $a \in A_C$ are apprehended and arrested with a fixed probability $p \in (0, 1)$. If arrested, they lose their legal income and payoff from crime and suffer from a punishment represented by $\psi \in \mathbb{R}_+$.

We can now specify the utility for each type and each strategy. Let $u : Q \times \mathbb{R} \to \mathbb{R}_+$ be a continuous increasing function representing utility from legal income Y_q and criminal payoff, strictly increasing in Y_q and S_a. We use the shorthand notation $u_q(a)$ for $u(Y_q, S_a)$ if the meaning is clear. Then

$$U_q(a, \chi_A) = \begin{cases} u(Y_q, 0) - H(\chi_A[q]) & \text{if } a = -1, \\ (1-p)u(Y_q, S[Y_a, \chi_A(a)]) - p\psi - H(\chi_A[q]) & \text{if } a \in A_C. \end{cases} \tag{1}$$

Thus, H represents disutility from crime in terms of units of utility from income and payoff.

By definition the density at every individual $q \in Q$ equals unity, thus we have $\chi_{\mathcal{U}}(U_q) = \int_{a \in A} \chi(U_q, a) = 1$ for all $U_q \in \mathcal{U}$ and $\chi_A(a) = \int_{U \in \mathcal{U}} \chi(U, a)\,\mathrm{d}U = \int_{q \in Q} \chi(U_q, a)\,\mathrm{d}q$.

Remark 2. *The disutility or harm H experienced by individuals due to crime does not depend on the success of their own criminal activity. We could consider a two-stage model in which individuals earn legal income and criminal payoff and experience disutility from crime in the first stage. In the beginning of the second stage, it is determined whether or not a criminal is arrested, and only individuals who are not arrested get to enjoy their utility u. The disutility H should be interpreted as consisting of psychological and collateral damage caused by victimization. For different specifications of U, individuals' choice of strategy could be influenced by the harm caused due to crime, i.e. individuals could prefer to choose $a \in A_C$ simply to escape the disutility H. Also note that, ceteris paribus, an increase in the crime density at q does not mean that more is stolen from that income type. Thus, we think of Y_q as "income net of theft".*

Let M denote a set $M = [m, 1] \subseteq Q$, $m \in [0, 1)$ such that $Y_m < Y_q$ for all $q \in (m, 1]$, and let $\bar{M} = Q \backslash M$, $A_M = M$, and $A_{\bar{M}} = \bar{M}$.

Assumption 2. *Strategy $a \in A_{\bar{M}}$ yields a payoff of zero: $S_a = 0$ for all $a \in A_{\bar{M}}$.*

Note that Assumption 2 is not a simplification of the model; in particular, $\bar{M} = \varnothing$ is possible.

So far, we have specified the primitives of the model in quite general terms, but we will impose several assumptions. The first two assumptions concern the crime payoff: Payoff is increasing and convex in the income of the target victim type, and decreasing and convex in the crime density at the target victim type; furthermore, every income in A_M yields enough payoff to "support" a positive measure of criminals targeting that type.

Assumption 3. *For all $a \in A_M$ and $Y_a > 0$, S_a satisfies the following conditions: $\partial S_a/\partial Y_a > 0$, $\partial^2 S_a/\partial Y_a^2 \geq 0$, $\partial S_a/\partial \chi_A(a) < 0$, $\partial^2 S_a/\partial \chi_A(a)^2 \geq 0$ with* $\lim_{\chi_A(a)\to\infty} S_a = 0$.

Assumption 4. *For every $\delta > 0$ and $a, a' \in A_M$, there exists an $\varepsilon \in (0, \delta)$, such that $S(Y_a, \varepsilon) > S(Y_{a'}, \delta)$ and $S(Y_a, \varepsilon) > 0$.*

We also adopt an assumption which guarantees that a set of individuals of measure greater than zero choose $a \in A_C$ in equilibrium, and that the crime density is strictly positive on M.

Assumption 5. *There exists an $\varepsilon > 0$ and a $q \in (0,1)$ such that for all $a \in A_M$,*

$$(1-p)u(Y_q, S[Y_a, \varepsilon]) - p\psi > u(Y_q, 0).$$

The next assumption concerns the disutility from crime.

Assumption 6. *If the set of individuals who choose strategy $a \in A_C$ has measure zero (i.e., $\chi_A(a) = 0$), individuals of income type a experiences no disutility: $H(a) = 0$. For all $a \in A_C$, H_a is a continuous and convave function, strictly increasing in $\chi_A(a)$: $\partial H_a/\partial \chi_A(a) > 0$, $\partial^2 H_a/\partial \chi_A(a)^2 \geq 0$.*

The function u is a quite general form for utility derived from legal income and criminal payoff. We will now adopt more assumptions about its form which are standard in economics.

Assumption 7. *We have $\partial u(y,S)/\partial y > 0$, $\partial^2 u(y,S)/\partial y^2 \leq 0$, $\partial u(y,S)/\partial S > 0$, $\partial^2 u(y,S)/\partial S^2 \leq 0$, $\partial^2 u(y,S)/\partial y \partial S \leq 0$, $\partial^2 u(y,S)/\partial S \partial y \leq 0$.*

We distinguish between assumptions and conditions. If a condition is assumed to hold, it will be explicitly noted.

Condition 1. *The function S satisfies $S(Y_a, \chi_A[a]) = R(Y_a)/\chi_A(a)$, where $R : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing in Y_a.*

Condition 2. *The function R is an affine transformation of Y: $R(y) = \beta y + \gamma$, with $\beta \in (0,1)$ and $\gamma \in \mathbb{R}$.*

Remark 3. *If $\gamma > 0$ we will necessarily have $S_a > 0$ for all $a \in A_C$, i.e. $m = 0$ and $M = \varnothing$.*

Condition 3. *The function u only depends on the sum of legal income and criminal payoff: $u(y, S) = u(y + S, 0)$.*

Condition 4. *The function H is an identity function, i.e. $H(\chi_a[q]) = \chi_a(q)$.*

2.3 *The Income Distribution and Inequality*

Given the definition of income distributions in Section 2.1, an income distribution $Y \in \mathcal{D}$ is already the inverse of a cumulative distribution function. Let $\bar{Y} = \int_{q \in Q} Y_q \, \mathrm{d}\, q$ denote the average income of a distribution. Then the *Lorenz curve* $Lo : \mathcal{D} \times Q \to [0,1]$ is given by (Gastwirth 1971)

$$Lo_Y(q) = \bar{Y}^{-1} \int_0^q Y(\tilde{q}) \, \mathrm{d}\, \tilde{q}. \tag{2}$$

The Lorenz curve can be used to define a partial ordering $\gtrsim_{Lo} \subseteq \mathcal{D} \times \mathcal{D}$, called the *Lorenz ordering*:

$$Y_1 \gtrsim_{Lo} Y_2 \text{ if } Lo_{Y_1}(q) \leq Lo_{Y_2}(q) \text{ for all } q \in Q. \tag{3}$$

The *Gini coefficient* $Gi : \mathcal{D} \to [0,1]$ is given by

$$Gi_Y = 1 - 2 \int_0^1 Lo_Y(\tilde{q}) \, \mathrm{d}\, \tilde{q}. \tag{4}$$

2.4 *Equilibrium*

We can now define the equilibrium of a game.

Definition 1. *For a game μ, a Borel measure χ on $\mathcal{U} \times A$ with marginals $\chi_{\mathcal{U}}$ and χ_A is an equilibrium crime distribution if we have*

(i) $\chi_{\mathcal{U}} = \mu$ *and*
(ii) $\chi(\{(U, a) : U(a, \chi_A) \geq U(a', \chi_A) \forall a' \in A\}) = 1.$

Remark 4. *What Definition 1 states is that (i) only crime distributions for which the marginal distribution on $\mathcal{U}$ corresponds to the game are considered, and that (ii) in equilibrium, every individual with any characteristics U who chooses strategy a has no incentive to deviate and choose a different strategy a', given the marginal distribution on A, i.e. given the strategy distribution of all individuals. Note that the equilibrium only considers the marginal distribution on A, irrespective of the characteristics of those who choose any particular strategy a. A more general definition would consider strategy profiles instead, i.e. a function from Q to A, or to a simplex of mixed strategies. As Roughgarden and Tardos (2004) note,*

> *[...] every action distribution is induced by some strategy profile. When all players select pure strategies, passing from strategy profiles to action distributions can be viewed as aggregating players according to their chosen strategies and ignoring their identities.*

The analysis in this section concerns only a distribution of strategy profile, because by definition criminal payoff from a strategy $a \in A_C$ only depends on the density of crime at the corresponding income type.

Proposition 1. *An equilibrium crime distribution exists.*

Remark 5. *Proposition 1 is a special case of very general theorems about the existence of equilibria in non-atomic games (see Schmeidler 1973, Mas-Colell 1984, Rath 1992). We therefore omit the proof, which would amount to a replication of, for example, Mas-Colell's proof. He shows (Mas-Colell 1984, Theorem 1) that for any game of the type given by Definition 1 an equilibrium distribution exists.*

Next, we will briefly summarize and proof some fairly obvious results.

Result 1. *If χ is an equilibrium crime distribution, then*

(i) $\chi_A(a) = 0$ *for* $a \in A_{\bar{M}}$,

(ii) $\chi_A(a) > 0$ *for all* $a \in A_M$,

(iii) $S_a = S_{a'}$ *for all* $a, a' \in A_M$.

Proof Suppose that χ is an equilibrium crime distribution, and

(i) $\chi_A(a) > 0$ for some $a \in A_{\bar{M}}$. Then $S_a = 0$ (by Assumption 2) and $S_{a'} > 0$ for some $a' \in A_M$ (by Assumption 3), thus $U_q(a, \chi_A) < U_Q(a', \chi_A)$ for all $q \in Q$ (by Assumption 7), a contradiction;

(ii) $\chi_A(a) = 0$ for all $a \in A_M$. Then there exists a $q \in (0,1)$ such that $U_q(-1, \chi_A) < U_q(a, \chi_A)$ for all $a \in A_M$ (by Assumption 5), a contradiction. Suppose that $\chi_A(a) = 0$ for some $a \in A_M$ and $\chi_A(a') > 0$ for some $a' \in A_M$. Then $S(Y_q, \varepsilon) > S(Y_q, \chi_A[a'])$ for some $\varepsilon > 0$ (by Assumption 4), thus $U_q(a, \chi_A) > U_q(a', \chi_A)$ (by Assumption 7), a contradiction;

(iii) $S_a > S_{a'}$ for some $a, a' \in A_M$. Then $U_q(a, \chi_A) > U_q(a', \chi_A)$ for all $q \in Q$ (by Assumption 7). With $\chi_A(a') > 0$ (by Result 1.[ii]), this is a contradiction.

■

Result 2. *In any equilibrium,* $\chi_A(a) > \chi_A(a')$ *for all* $Y_a > Y_{a'}$, $a, a' \in A_M$.

Proof Suppose χ is an equilibrium crime distribution and $\chi_A(a) \leq \chi_A(a')$ for some $a > a'$ and $Y_a > Y_{a'}$. Then $S(Y_a, \chi_A[a]) > S(Y_{a'}, \chi_A[a])$ (by Assumption 3). By Result 1(iii), this is a contradiction. ■

The next result implies that there is "marginal criminal", and that only the poorest players become criminals.

Result 3. *Suppose χ is an equilibrium crime distribution. Then*

$$\int_{a \in A_C} \chi(U_q, a)\, \mathrm{d}\, a > 0 \Rightarrow \int_{a \in A_C} \chi(U_{q'}, a)\, \mathrm{d}\, a > 0 \;\; \textit{for all } q > q',$$

$$\int_{a \in A_C} \chi(U_q, a)\, \mathrm{d}\, a = 0 \Rightarrow \int_{a \in A_C} \chi(U_{q'}, a)\, \mathrm{d}\, a = 0 \;\; \textit{for all } q < q'.$$

Proof Recall the shorthand $u_q(a) = u(Y_q, S_a)$. We have

$$U_q(a, \chi_A) \geq U_q(-1, \chi_A) \Leftrightarrow (1-p)u_q(a) - p\,\psi \geq u_q(-1),$$

for all $a \in A_C$; the equivalence also hold for strict inequality.

Lemma 1. *The following statements are true in equilibrium:*

$$\int_{a \in A_C} \chi(U_q, a) > 0 \Rightarrow (1-p)u_q(a) - u_q(-1) \geq p\,\psi, \tag{5a}$$

$$\int_{a \in A_C} \chi(U_q, a) = 0 \Rightarrow (1-p)u_q(a) - u_q(-1) \leq p\,\psi. \tag{5b}$$

Proof of Lemma 1 Suppose in equilibrium $\int_{a \in A_C} \chi(U_q, a) > 0$ and $(1-p)u_q(a) - u_q(-1) < p\,\psi$ for some $a \in A_M$; then $U_q(-1, \chi_A) > U_q(a, \chi_A)$, a contradiction. Suppose in equilibrium $\int_{a \in A_C} \chi(U_q, a) = 0$ and $(1-p)u_q(a) - u_q(-1) < p\,\psi$ for some $a \in A_M$; then $U_q(-1, \chi_A) < U_q(a, \chi_A)$, a contradiction. □

Lemma 2. *The following statements are true in equilibrium:*

$$(1-p)u_q(a) - u_q(-1) \geq p\,\psi \Rightarrow (1-p)u_{q'}(a) - u_{q'}(-1) > p\,\psi \quad \textit{for all } q > q', \tag{6a}$$

$$(1-p)u_q(a) - u_q(0) \leq p\,\psi \Rightarrow (1-p)u_{q'}(a) - u_{q'}(-1) < p\,\psi \quad \textit{for all } q < q'. \tag{6b}$$

Proof of Lemma 2 It is sufficient to show that in equilibrium $(1-p)u_q(a) - u_q(-1) < (1-p)u_{q'}(a) - u_{q'}(-1)$ for all $q > q'$, which is equivalent to $(1-p)[u_q(a) - u_{q'}(a)] < u_q(-1) - u_{q'}(-1)$. Because $p \in (0,1)$, it sufficient that $u_q(a) - u_{q'}(a) \leq u_q(-1) - u_{q'}(-1)$. By Assumption 7 we have

$\partial u(Y, S_a)/\partial Y \leq \partial u(Y, 0)/\partial Y$ for $S_a > 0$. We have $S_a = S_{a'} > 0$ for all $a \in A_M$ (by Assumption 5 and Result 1), and with $q < q'$ we have $Y_q \leq Y_{q'}$, which proves Eq. (6b). The proof works analogously for Eq. (6a). □

Lemmata 1 and 2 together prove Result 3. ■

Result 3 shows that there is a *marginal criminal* of some income type $q^* \in Q$; individuals of type $q < q^*$ with income $Y_q \leq Y_{q^*}$ choose $a \in A_C$ and individuals of type $q > q^*$ with income $Y_q \geq Y_{q^*}$ choose $a = -1$. The previously defined crime rate $c^* = 1 - \chi_A(-1)$ is then equal to q^*. It can easily be seen that $U_{q^*}(a, \chi_A) = U_{q^*}(-1, \chi_A)$ if χ_A is an equilibrium crime distribution.

Informally, rising inequality is interpreted as "the rich get richer" and "the poor get poorer". Because lower income of "poor" individuals will increase the difference of the utilities from choosing $a \in A_C$ and $a = -1$, i.e. $(1 - p)u_q(a) - u_q(-1)$, it seems natural that rising inequality increases the crime rate. This is not necessarily the case, as we have to consider the legal income of the marginal criminal. The next result illustrates this.

Result 4. *For two income distributions Y_1 and Y_2, $Y_1 \gtrsim_{Lo} Y_2$ does not necessarily imply $c_1^* \geq c_2^*$. It is possible that $Y_1 \gtrsim_{Lo} Y_2$ and $c_1^* < c_2^*$. As a corollary, a higher Gini coefficient does not necessarily imply a higher crime rate.*

Proof Consider Figure 1, where $M = Q$. The equilibrium crime level for the old linear income distribution is indicated by c_1^*. Now a shift of income incurs such that $Y_{q,2} < Y_{q',1}$ for some $q' < q_1^*$ and all $q \leq q'$, and $Y_{q,2} > Y_{q',1}$ for all $q > q'$ and $Y_{q',2} = Y_{q',1}$ and unchanged average and total income. It can be easily seen that $Y_1 \gtrsim_{Lo} Y_2$.

We have $Y_{q_1^*,2} > Y_{q_1^*,1}$. Suppose for simplicity that $S(Y_a, \chi_A[a]) = \beta Y_a/\chi_A(a)$ with $\beta \in (0,1)$ (i.e. Conditions 1 and 2 hold with $\gamma = 0$) and $u(Y_q, S_a) = Y_a + S_a$ for all $a \in A_M = A_C$ and all $U \in \mathcal{U}_{Y_i}$, $i \in \{1,2\}$. There exist $\beta, p \in (0,1)$ and $\psi \geq 0$ such that individual of income type q_1^* is indeed the marginal criminal for the income distribution Y_1. Given the assumptions on S_a and u_q above, $S_{a,1} = S_{a,2}$ if $q_1^* = q_2^*$ because total income remains unchanged. But for equilibrium crime distributions $\chi_{A,1}$ and $\chi_{A,2}$ we must have $(1-p)u(Y_{q_i^*,i}, S_{a,i}) - p\psi = u(Y_{q_i^*,i}, 0)$ for all $a \in A_M$ and $i \in \{1,2\}$. If $q_2 = q_1$, then $S_{a,1} = S_{a,2} = S_a$ for all $a \in A_M$ and with $Y_{q_2^*,2} > Y_{q_2^*,1}$ we have $(1-p)u(Y_{q_2^*,2}, S_a) - p\psi < u(Y_{q_2^*,2}, 0)$, a contradiction. With Result 3 it follows that $q_2^* < q_1^*$ in equilibrium. ■

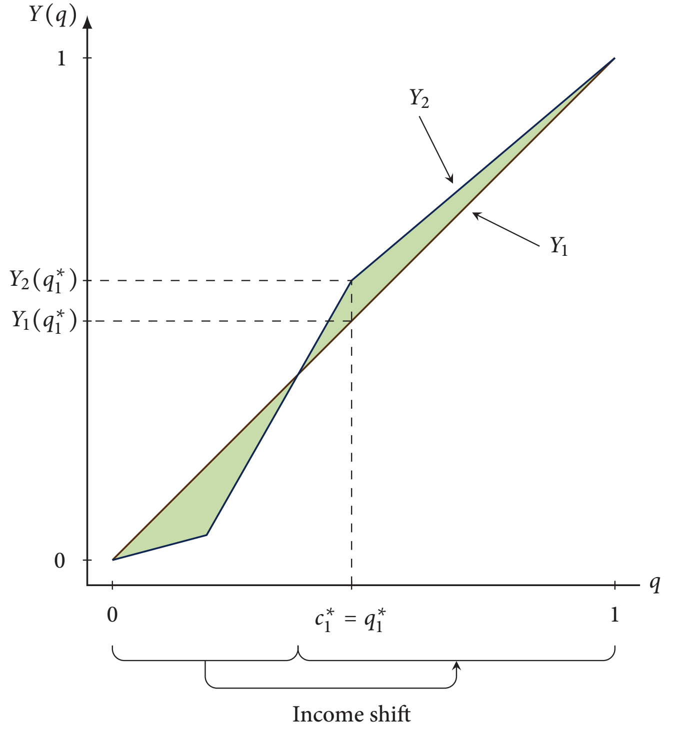


Figure 1: A less equal income distribution can lead to a lower crime rate, even if it Lorenz-dominates.

Remark 6. *Assuming $S(Y_a, \chi_A[a]) = \beta Y_a/\chi_A(a)$ and $u(Y_q, S_a) = Y_a + S_a$ is not necessary for Result 4, but it simplifies the proof. The intuition for the result is simple: The income shift increases incentives for individuals of income type $q < q'$ to choose crime, but all individuals of income type $q < q'$ are already criminals, thus an increase in the crime rate cannot come from these individuals. The increase in income of income type q_1^* decreases the incentives* *of that income type to choose crime. Overall, the aggregated "stealable" income remains constant. The result highlights the important feature that it is changes in incentives of the marginal criminal which lead to changes in the crime rate.*

2.5 *Further Results*

It light of Result 4, we would like to know under what circumstances a change in the income distribution leads to a higher crime rate. In general, we can distinguish two effects of changes of the income distribution: The effect on the legal income of individuals at the margin between choosing crime and no crime, and the effect on the payoff to crime. Because the crime rate is determined by the q which solves $(1-p)\,u(Y_q, S) - p\,\psi = u(Y_q, 0)$, an increase in Y_q decreases the incentives to become criminal, and an increase in overall income can (but not necessarily does) increase S, leading to an increase in the incentives to become criminal. The latter effect is not necessarily positive because if income is partly shifted from individuals from which is it more efficient to steal (with high $\partial S/\partial Y$) to individuals from which is it less efficient (individuals in $[0, m]$ or with low $\partial S/\partial Y$), total "stealable income" actually decreases. Thus, to avoid this problem and simplify the analyis, we will invoke Conditions 1 and 2.

If Condition 1 holds, we define $\bar{S}$ as

$$\bar{S} = (q^*)^{-1} \int_{\tilde{q}\in Q} R(Y_{\tilde{q}})\,\mathrm{d}\,\tilde{q};$$

note that $\bar{S}$ is independent of q.

Result 5. *Suppose Condition 1 holds. Then in equilibrium* $\bar{S} = S_a$ *and* $\chi_A(a) = \bar{S}^{-1}\,R(a)$ *for all* $a \in A_M$.

Proof With

$$S_a = \frac{R(Y_a)}{\chi_A(a)} = \frac{\int_{\tilde{a}\in A_C} R(Y_{\tilde{a}})\,\mathrm{d}\,\tilde{a}}{\int_{\tilde{a}\in A_C} \chi_A(\tilde{a})\,\mathrm{d}\,\tilde{a}}$$

and $q^* = \int_{\tilde{a}\in A_C} \chi_A(\tilde{a})\, \mathrm{d}\,\tilde{a} = \int_{\tilde{a}\in A_M} \chi_A(\tilde{a})\, \mathrm{d}\,\tilde{a}$, the result follows immediately. ■

The result states that in equilibrium, the crime distribution is determined by R up to a multiplicative constant (given the overall crime level).

Result 6. *Suppose Conditions 1 and 2 hold, with $\gamma = 0$ and $m = 0$. Let Y_1 and Y_2 be two different income distributions and let $c_1^* = q_1^*$ and $c_2^* = q_2^*$ denote the respective crime rates; $Y_{q,1}$ and $Y_{q,2}$ denote the incomes of type q for the two distributions. Then in equilibrium, $c_1^* < c_2^*$ if*

(i) $Y_{q,1} < Y_{q,2}$ for all q in some non-empty interval $[q_1, q_2] \subset Q$ with $q_1^* \notin [q_1, q_2]$ and $Y_{q,1} = Y_{q,2}$ for all other $q \notin [q_1, q_2]$; or, more generally, for a set of non empty intervals, if

$$\begin{aligned} &Y_{q,1} < Y_{q,2} \ \text{ for all } \ q \in [q_i, q_{i+1}] \subseteq Q \ \text{ for some } \ i = 1, \dots, n, \\ &\text{with } \ q_1^* \notin [q_i, q_{i+1}] \ \text{ for all } \ i = 1, \dots, n \\ &\text{and } \ Y_{q,1} = Y_{q,2} \ \text{ for all other } q \end{aligned}$$

(ii) $Y_{q,1} > Y_{q,2}$ for all q in some non-empty interval $[q_1^* - \varepsilon_1, q_1^* + \varepsilon_2] \subset Q$ for sufficiently small ε_1, ε_2.

Proof

(i) Suppose $c_1^* \geq c_2^*$. We have $S_{a,1} < S_{a,2}$ and $u(Y_{q_1^*,1}, 0) = u(Y_{q_1^*,2}, 0)$, a contradiction.

(ii) Suppose $c_1^* = c_2^*$, and we have $S_{a,1} > S_{a,2}$. Thus

$$u(Y_{q_1^*,1}, S_{a,1}) - u(Y_{q_1^*,2}, S_{a,2}) > u(Y_{q_1^*,1}, 0) - u(Y_{q_1^*,2}, 0),$$

but for small enough ε_1 and ε_2, we have

$$(1-p)\,[u(Y_{q_1^*,1}, S_{a,1}) - u(Y_{q_1^*,2}, S_{a,2})] - p\,\psi > u(Y_{q_1^*,1}, 0) - u(Y_{q_1^*,2}, 0).$$

■

Another interesting result is that if an increase in income of some individuals of measure greater than zero leads to increase in the crime rate, it will increase the overall crime less than the crime density at these types, thus reducing the crime density at other types.

Result 7. *Suppose Conditions 1 and 2 hold, with* $\gamma = 0$ *and* $m = 0$*. Suppose for simplicity that* Y_q *is strictly increasing in* q*. If the income of types in some non-empty interval* $[q_1, q_2] \subset (q_1^*, 1]$ *increases, then* $c_2^* > c_1^*$, $\chi_{A,1}(q) > \chi_{A,2}(q)$ *for all* $q \in [q_1, q_2]$*, and* $\chi_{A,1}(q) < \chi_{A,2}(q)$ *for all* $q \in Q \backslash [q_1, q_2]$*, where the subscripts* 1 *and* 2 *denote the situations before and after the change of the income distribution, respectively.*

Proof The first part, $c_2^* > c_1^*$, is the same as Result 6(i). By Result 5 and with Condition 2 and $\gamma = 0$, we have $\chi_A(a) = \bar{S}^{-1} \beta Y_a$. Thus, if $\bar{S}$ increases and Y_a remains constant (for $a \in Q \backslash [q_1, q_2]$), then the result follows. We have

$$\bar{S} = (q^*)^{-1} \beta \int_{\tilde{q} \in Q} Y_{\tilde{q}} \, \mathrm{d} \, \tilde{q}.$$

Suppose after the change in the income distribution, $\bar{S}$ remains constant or decreases. By Result 5, $\bar{S} = S_a$. Then $u(Y_{q_1^*}, S_{a,1}) \geq u(Y_{q_1^*}, S_{a,2})$, implying $q_1^* > q_2^*$, a contradiction. ∎

We summarize the main results of this section in the following (informal) proposition:

Proposition 2. *There exists an equilibrium crime distribution which equalizes payoff of stealing from any income type for which stealing is profitable at all. There exists a marginal criminal: All individuals with lower income supplement their income with crime, while all individuals with higher income do not commit crimes. Inequality tends to increase crime by (i) lowering the income of poorer individuals and/or (ii) increasing "stealable income"; however, for (i) it matters whether the income of the marginal criminal is among the incomes which decrease; it is possible to find examples for which higher inequality actually leads to less crime. Higher income of high income types can decrease the harm* *from crime suffered by poor and intermediate income types: Even though crime increases, the crime targeted at high income types increases by more than overall crime, thus shifting some of the crime from poor and intermediate types to the rich.*

2.6 *An Example*

A short example based on the Singh-Maddala (1976) income distribution, which is given by

$$F^S(y) = 1 - \frac{1}{(1 + [x/\alpha_2]^{\alpha_1})^{\alpha_3}}, \tag{7}$$

may be helpful to understand the basic model. We will only focus on a one-parameter case, namely $F^S_\alpha(y) = 1 - 1/(1 + x^\alpha)$. Using a result due to Wilfling and Krämer (1993), we have $F^S_\alpha(y) \gtrsim_{Lo} F^S_{\alpha'}(y)$ if and only if $\alpha \leq \alpha'$. For $F^S_\alpha(y)$, we have

$$Y_q = \left(\frac{q}{1-q}\right)^{1/\alpha}. \tag{8}$$

To keep the example simple, we assume that $u(y, S) = y + S$, $S_a = \beta\, Y_a/\chi_A(a)$, $\psi = 0$, and $M = \{\}$. Then the aggregate amount of criminal payoff is simply given by $\int_{a\in A_C} \beta\, Y_a \,\mathrm{d}\, a$, and

$$\bar{S} = (q^*)^{-1} \int_{a\in A_C} \beta\, Y_a \,\mathrm{d}\, a. \tag{9}$$

Using Result 5, we have that $S_a = \bar{S}$ for all $a \in A_C$, and we know that the the marginal criminal q^* is given by the q which solves

$$(1-p)\left(Y_q + q^{-1} \int_{a\in A_C} \beta\, Y_a \,\mathrm{d}\, a\right) = Y_q. \tag{10}$$

The crime distribution can be computed as

$$\chi_A(a) = \frac{q^* \, Y_a}{\int_{a \in A_C} Y_a \,\mathrm{d}\, a}. \tag{11}$$

Figure 2 shows the crime distribution for different α.

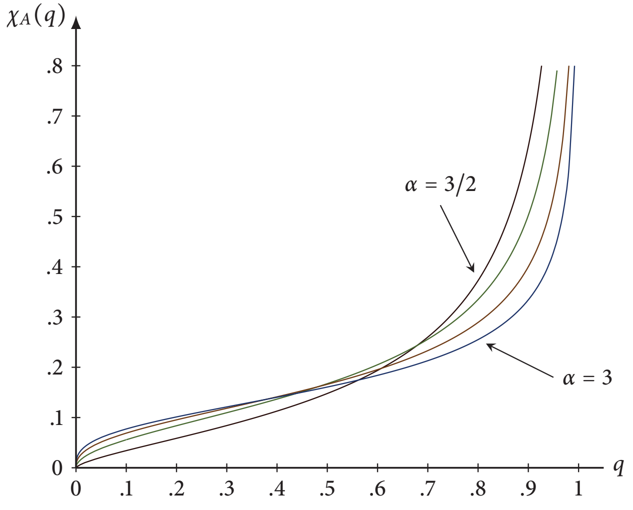


Figure 2: The equilibrium crime density for $\alpha \in \{3/2, 2, 5/2, 3\}$ without private security ($\beta = 1/10$, $p = 1/2$).

2.7 *Discussion*

The result that, loosely speaking, "all of the poor and none of the rich are criminals", is of course a result of the simplifications of the model and not realistic. One the one hand, the model does not consider white collar crimes like tax fraud. On the other hand, the model assumes complete homogeneity of attitudes towards crime.[3] An obvious extension of this section would be to add a dimension to the type space to account for heterogeneous "corruptedness" of

[3] While the utility U depends on income (thus type) of an individual, the utility function u has the same form for each individual of every income type.

individuals. In the simplest case, we could consider two sub-populations such that all members of one sub-population consider crime as a possible option without any moral concerns, whereas members of the other sub-population are incorruptible and exclude crime from their considerations; these individuals would be dummy players with a strategy space consisting only of the strategy -1. A more sophisticated approach would consider an additional continuous type-dimension, e.g. $Z = [0,1]$ such that payoff from crime is discounted according to $z \in Z$; say, $u(y,S) = y + zS$. The (marginal) distribution on Z could also depend on the income type. Another generalization would be to consider a correlation between legal income opportunity and payoff from crime; if skill and education determine legal income, these factors could also affect illegal income.

Some of the assumptions made so far deserve at least a brief discussion. The assumptions that payoff from crime cannot be stolen again and that disutility from crime does not involve loss of income are obviously unrealistic. Remark 1 briefly gives a reason for the first assumption. Remark 2 explains the concept of harm from crime and that we think of income Y as "net of theft". Note that payoff from crime will be equal for all individuals who decide to become criminals. If, say, $u(y,S) = v(y+S)$, with v being a usual Bernoulli utility function, then crime will somewhat raise and equalize total income among lower income types. If that income could be stolen again, some crime would shift from higher income types to the lower (criminal) income types. If loss of income due to theft would also decrease utility from income, crime may either increase marginally (due to higher incentives via loss of income) or decrease marginally (due to lower extra income from crime and the increased harm that comes with increased income). Overall, the major results would remain qualitatively unchanged.

Another assumptions which simplifies the model is that legal income is supplemented by criminal income. And alternative and perhaps more usual setup would have individuals decide how much of their time to devote to legal work and crime, which would also add another dimension to the strategy space. However, with the assumption that criminal payoff cannot be stolen, individuals could then prefer crime over legal work partly because it reduces the harm suffered from crime (see also Remark 2). The current specification avoids this problem. The model still produces the intuitive result that a higher legal income reduces incentives to become criminal because of increasing opportunity costs (i.e. legal income) and decreasing returns to crime.

3 PRIVATE SECURITY

3.1 *Preliminaries*

After the basic model has been set up in Section 2, we will now introduce a new type of player, a *private security company* (PSC). We let $n \in \mathbb{N}$ be the total number of PSCs on the market and denote $N = \{1, \ldots, n\}$. A PSC $\ell \in N$ offers a certain amount of *effort* $e_\ell \in \mathbb{R}_+$ against crime, which he tries to sell at a price $\rho_\ell \in \mathbb{R}_+$. For the provision of an amount e_ℓ of effort, the PSC incurs costs, given by a continuous and strictly increasing variable cost function $k_\ell : \mathbb{R}_+ \to \mathbb{R}_+$ with $k_\ell(0) = 0$. We denote $\mathbf{e} = (e_1, \ldots, e_\ell)$ the vector of efforts provided by the PSCs. With $\bar{n} \geq 1$ PSCs, we denote $e_\Sigma = \sum_{\ell \in N} e_\ell$ the aggregate effort provided. We assume that the effort provided by the PSCs is perfectly substitutable, i.e. there is no difference in quality of effort. PSCs engage in Cournot competition, thus selling at the same market clearing price $\rho_\ell = \rho$. Thus, their characterizing profit function $\pi : \mathbb{R}^2_+ \to \mathbb{R}$ is given by

$$\pi_\ell(e_\ell, \rho) = e_\ell \, \rho - k_\ell(e_\ell) \tag{12}$$

Let $\Pi = \{\pi_\ell\}_{\ell=1}^n$ denote the set of profit functions of all PSCs.

Let $X = \mathbb{R}_+ \cup \{\infty\}$. An *effort allocation* is a Lebesgue measurable and integrable function $\xi : Q \to X$ that satisfies $\int_{q \in Q} \xi(\tilde{q}) \, \mathrm{d}\, \tilde{q} \leq e_\Sigma$. Let Ξ be the set of all effort allocations.

Each individual in characterized by utility function $V : A \times \mathcal{M} \times X \times \Xi \to \mathbb{R}$. Given the strategy $a_q \in A$, the strategy distribution ν of all individuals, and the effort allocation ξ, $V_q(a_q, \nu, \xi(q), \xi)$ represents the utility of that individual. Let $\mathcal{V}$ be the set of all utility functions. The economy is then characterized by (μ, Π), where μ is a (Borel) measure on $\mathcal{V}$, i.e. a game of crime.

3.2 *The Utility Functions*

As before, the payoff from stealing of individuals of income type q depends on their income Y_q and the crime density at q, but now it also depends on $\xi(q)$. This payoff can be represented by a continuous function $S : Q \times \mathbb{R}_+ \times X \to \mathbb{R}_+$.

We assume that the effort against crime purchased will not directly reduce the disutility from criminal activity. It can, however, change the disutility indirectly via a different crime density induced by ξ. We will model the effect of ξ as a transformation of the income that can be stolen; more about this Assumption 3' below.

We can now specify the utility for each type and each strategy.

$$V_q(a, \chi_A, x, \xi) = \begin{cases} u(Y_q - \rho x, 0) - H(\chi_A[q]) & \text{if } a = -1, \\ (1-p)u(Y_q - \rho x, S[Y_a, \chi_A(a), \xi(a)]) & \\ \qquad\qquad -p\psi - H(\chi_A[q]) & \text{if } a \in A_C. \end{cases} \tag{13}$$

We assume that individuals cannot spend more than their income Y_q on effort against crime, but this is an economic restraint, not a technical one. We adopt all of the assumptions specified in Section 2.2, which easily generalize to the case considered here, except for Assumption 3, for which we adopt Assumption 3'. Let $T : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$ be a continuous measurable integrable function.

Assumption 3'. *It holds that* $S(Y_a, \chi_A[a], \xi[a]) = S(T[Y_a, \xi(a)], \chi_A[a])$, *where* T *is strictly increasing in* Y_a *and strictly decreasing in* $\xi(a)$ *for all* $a \in A_M$, *with* $T(y, 0) = y$, $T(y, x) < y$ *for* $x > 0$ *and* $T(y, x) > y$ *for* $x < 0$. *Furthermore,* T *is convex in* $\xi(a)$: $\partial T(y, x)/\partial y > 0$, $\partial T(y, x)/\partial x < 0$ *and* $\partial^2 T(y, x)/\partial x^2 > 0$, *and* $\lim_{x\to\infty} T(y, x) \to Y_m$ *if* $\bar{M}$ *is not empty and* $\lim_{x\to\infty} T(y, x) \to Y_0$ *otherwise. Furthermore,* $\partial^2 T(y, x)/(\partial x \partial y) > 0$.

For all $a \in A_M$ and $Y_a > 0$, S_a satisfies the following conditions: $\partial S_a / \partial T_a > 0$, $\partial^2 S_a / \partial T_a^2 \geq 0$, $\partial S_a / \partial \chi_A(s) < 0$, $\partial^2 S_a / \partial \chi_A(a) \geq 0$ with $\lim_{\chi_A(a) \to \infty} S_a = 0$.

Remark 7. *The function T transforms the income distribution Y into a new distribution, in a sense reducing the "stealable income", and can be interpreted as a protection technology with decreasing marginal returns.*

Assumption 3' gives very general conditions for the transformation of income. One particular form which satisfies the assumptions is given in Condition 5.

Condition 5. *The function T satisfies $T(y, x) = b^{-tx} y$, with $b > 1$ and $t > 0$.*

3.3 *Equilibrium*

We can now define the equilibrium of the economy. Let τ be a Borel measure on $\mathcal{V} \times X$.

Definition 2. *For an economy (μ, Π), a tuple (χ, ξ, ρ, e) is a competitive crime-security-equilibrium if*

(i) χ *is an equilibrium crime distribution of the game of crime* μ, *i.e.* $\chi_{\mathcal{V}} = \mu$ *and* $\chi(\{(V, a) : V(a, \chi_A, \xi(a), \xi) \geq V(a', \chi_A, \xi(a), \xi) \forall a' \in A\}) = 1$.

(ii) $\tau(\{(V, x) : V(a, \chi_A, x, \xi) \geq V(a, \chi_A, x', \xi) \forall x' \in X$ *and* $\rho x, \rho x' \leq Y\}) = 1$.

(iii) $\int_{q \in I} \xi(\tilde{q}) \, \mathrm{d} \tilde{q} = e_\Sigma$,

(iv) $\pi_\ell(e_\ell, \rho) \geq \pi_\ell(e, \rho)$ *for all* $e \in \mathbb{R}_+$ *and all* $\ell \in N$.

The effort allocation ξ is a symmetric equilibrium effort allocation if additionally we have

$$\tau\left(\{(V_q, \xi[q]) : q \in Q\}\right) = 1$$

i.e. individuals of the same income type choose the same effort.

Proposition 3. *A competitive crime-security-equilibrium with a symmetric equilibrium effort allocation exists.*

Proof We first establish the following easy lemma:

Lemma 3. *For any given symmetric effort allocation ξ, there exists an equilibrium crime distribution χ^{ξ}.*

Proof of Lemma 3 Given the assumptions on T (Assumption 3'), any effort allocation $\hat{\xi}$ transforms Y, leading to a new distribution relevant for the choice of a. It can be easily seen that Proposition 1 holds as before. □

The next lemma, while based on a result due to Rath et al. (1995), is also quite simple (see Remark 8 below).

Lemma 4. *For any given price $\rho > 0$, there exists ξ such that every individual of type q chooses $\xi(q)$ and*

$$\tau\Big(\Big\{(V,x) : V(a,\chi_A,x,\xi) \geq V(a,\chi_A,x',\xi) \text{ for all } x' \in X \text{ and } \rho\, x, \rho\, x' \leq Y\Big\}\Big) = 1 \quad (14)$$

$$\tau\Big(\Big\{(V_q, \xi[q]) : q \in Q\Big\}\Big) = 1 \quad (15)$$

That is, there exists a symmetric equilibrium effort allocation.

Proof of Lemma 4 The best reply correspondence $B : \mathcal{V} \to X$ is given by

$$B(V) = \{x \in X : V(a,\chi_A,x,\xi) \geq V(a,\chi_A,x',\xi) \forall x' \in X\}.$$

Note that Eq. (14) is equivalent to

$$\partial u(Y_q - \rho\, \xi[q], S_a)/\partial\xi(q) = \partial H(\chi_A[q])/\partial\xi(q) \quad (16a)$$
$$\text{whenever } \xi(q) > 0 \text{ and } \rho\, \xi(q) \leq Y_q,$$

$$\partial u(Y_q - \rho\, \xi[q], S_a)/\partial\xi(q) \leq \partial H(\chi_A[q])/\partial\xi(q) \quad (16b)$$
$$\text{whenever } \xi(q) > 0 \text{ and } \rho\, \xi(q) = Y_q,$$

$$\partial u(Y_q - \rho\, \xi[q], S_a)/\partial\xi(q) \geq \partial H(\chi_A[q])/\partial\xi(q) \quad (16c)$$
$$\text{whenever } \xi(q) = 0.$$

It can be easily seen that given ρ and ξ, the best reply of any individual is a singleton. Thus, B is single-valued. Now, let

$$\mathcal{X}_B = \{\mu \circ b^{-1} : b \text{ is a measurable selection from } B\}.$$

Rath et al. (1995) show that if $\mathcal{X}_B$ is closed, a symmetric equilibrium exists. Because B is single-valued, we have that

$$\{b : b \text{ is a measurable selection from } B\} = B,$$

so $\mathcal{X}_B$ is trivially closed. □

Given the existence of an equilibrium crime distribution and an equilibrium effort allocation for given ρ, a demand or inverse demand function for effort exists. Standard arguments then easily establish an equilibrium vector **e**. ■

Remark 8. *Lemma 4 is a special case of Theorem 4 in Rath et al. (1995). The existence of an equilibrium allocation follows directly from existence theorems for non-atomic games (Schmeidler 1973, Mas-Colell 1984, Rath 1992). However, for compact but uncountably infinite strategy sets there may not exist a symmetric equilibrium distribution, i.e. an equilibrium where all individuals of the same type choose the same strategy. This was not relevant for Proposition 1, as we were only interested in the crime density at q without regard to which of the income types stole from q. Now the distribution matters, as the effect of an amount of effort x can depend on the type who chooses this x.*

Lemma 4 is not surprising at all. Ultimately (and informally) a multi-valued best reply is a necessary condition for the non-existence of a symmetric equilibrium. The conditions for an equilibrium in the case considered here imply that the best reply is always single-valued, so the problem disappears completely. In fact, we know from the literature that an equilibrium effort allocation exists, and the conditions for such an equilibrium derived from this model imply that if an equilibrium exists, it has to be symmetric.

Result 8. *In any competitive crime-security-equilibrium, for* $q \in \bar{M}$, $\xi(q) = 0$. *For any given* e_Σ *and all* $q, q' \in M$, $\xi(q) \leq \xi(q')$ *whenever* $Y_q < Y_{q'}$ *and* $\xi(q') \geq 0$.

Proof The first statement is obvious and follows from Result 1.(i). The second statement follows from

$$\left.\frac{\partial u(Y_q - \rho\,\xi[q])}{\partial \xi(q)}\right|_{\xi(q)=\hat{x}} \leq \left.\frac{\partial u(Y_{q'} - \rho\,\xi[q'])}{\partial \xi(q')}\right|_{\xi(q')=\hat{x}} \quad \text{for all } \hat{x} \in X \text{ and } Y_q < Y_{q'} \tag{17}$$

and

$$\left.\frac{\partial H(\chi_A[q])}{\partial \xi(q)}\right|_{\xi(q)=\hat{x}} > \left.\frac{\partial H(\chi_A[q'])}{\partial \xi(q')}\right|_{\xi(q')=\hat{x}} \quad \text{for all } \hat{x} \in X \text{ and } Y_q < Y_{q'}. \tag{18}$$

Eq. (17) is obvious and follows from Assumption 7. Eq. (18) follows from $\partial^2 T(y,x)/\partial x^2 > 0$ and $\partial^2 S_a/\partial T_a^2 \geq 0$ (by Assumption 3') and Result 2 which states that $\chi_A(a) < \chi_A(a')$ if $Y_a < Y_{a'}$, which can be transferred to the framework here, i.e. $\chi_A(a) < \chi_A(a')$ if $T_a < T_{a'}$. ∎

Result 8 states that income types which do not attract any crime at all ($q \in M$) will not demand any effort against crime in equlibrium. Furthermore, demand for effort weakly increases in income.

3.4 *Further Results*

Given the basic model in the previous section, we now turn to the implications for the distribution of crime with private security.

Result 9. *It is possible that "intermediate" income types suffer from a higher crime density than "rich" individuals: Suppose demand for effort against crime is positive for a set of individuals of measure greater than zero, and Conditions 1, 2, 3, 4, and 5 hold with* $\gamma = 0$, *and that* $M = \varnothing$. *Then*

1. *if utility from income is linear, either*
    - $\xi(q) > 0$ *and* $\chi_A(q) = \chi_A(q')$ *for all* $q, q' \in Q$, *or*
    - *there exists* $q_0 \in Q$ *such that* $\xi(q) = 0$ *for all* $q \in [0, q_0]$, $\xi(q) > 0$ *for all* $q \in (q_0, q]$, *and* $\chi_A(q) = \chi_A(q') < \chi_A(q_0) = \chi_A(q'')$ *whenever* $Y_q < Y_{q'}$ *for* $q, q' \in [0, q_0)$ *and* $q'' \in (q_0, 1]$;
2. *if utility from income is strictly concave, either*
    - $\xi(q) > 0$ *for all* $q \in Q$ *and* $\chi_A(q) > \chi_A(q')$ *whenever* $Y_q < Y_{q'}$, *or*
    - *there exists* $q_0 \in Q$ *such that* $\xi(q) = 0$ *for all* $q \in [0, q_0]$, $\xi(q) > 0$ *for all* $q \in (q_0, q]$, *and* $\chi_A(q) < \chi_A(q')$ *whenever* $Y_q < Y_{q'}$ *for* $q, q' \in [0, q_0]$, *and* $\chi_A(q) > \chi_A(q')$ *whenever* $Y_q < Y_{q'}$ *for* $q, q' \in [q_0, 1]$.

Proof The proof is based on the following lemma:

Lemma 5. *Suppose Condition 5 holds. Then*

$$\frac{\partial T(y,x)}{\partial x} = \frac{\partial T(y',x')}{\partial x'} \Leftrightarrow T(y,x) = T(y',x'). \tag{19}$$

If additionally Conditions 1 and 2 hold, then in equilibrium we must have

$$\frac{\partial \chi(q)}{\partial \xi(q)} = \frac{\partial \chi(q')}{\partial \xi[q']} \Leftrightarrow \chi(q) = \chi(q'). \tag{20}$$

If Conditions 1, 2, 4, and 5 hold, then in equilibrium we must have

$$\frac{\partial H(\chi[q])}{\partial \xi(q)} = \frac{\partial H(\chi[q'])}{\partial \xi[q']} \Leftrightarrow H(\chi[q]) = H(\chi[q']). \tag{21}$$

Proof of Lemma 5 With $T(y,x) = b^{-tx}y$ (Condition 5), we obtain $\partial T(y,x)/\partial x = -\log(b)b^{-tx}y$. Then $\partial T(y,x)/\partial x = \partial T(y',x')/\partial x'$ is equivalent to $b^{-tx}y = b^{-tx'}y'$, and Eq. 19 follows. With $S(T[Y_q, \xi(q)], \chi_A[q]) = \beta\, T[Y_q, \xi(q)]/\chi_A(q)$ (Conditions 1 and 2) and $S_q = S_{q'}$ (Result 1) it follows that $\chi_A(q) = \chi_A(q')$ if and only if $T(Y_q, \xi[q]) = T(Y_{q'}, \xi[q'])$. Then with Eq. 19, Eq. 20 follows. Finally, Eq. 21 is obvious. □

1. We have $\partial u(Y_q - \rho x)/\partial x = -\rho$. Thus in equilibrium $-\partial H(\chi_A[q])/\partial x = \rho$ for all $q \in Q$ such that $\xi(q) > 0$ (i.e. all $q \in [q_0, 1]$). With Lemma 5 it follows that $\chi(q) = \chi(q')$.
2. In equilibrium $-\partial H(\chi_A[q])/\partial x = \partial u(Y_q - \rho x, S_a)/\partial x$; $\partial u(Y_q - \rho x, S_a)/\partial x$ is decreasing in Y_q, implying $\chi_A(q) < \chi_A(q')$ whenever $Y_q < Y_{q'}$ for all $q, q' \in [q_0, 1]$.

■

Remark 9. *Result 9 also holds for somewhat less restrictive assumptions. What is particularly interesting about the result is that with linear utility, private security equalizes harm (and thus, crime) among those who purchase any positive amount of effort against crime, and that with decreasing marginal utility, the crime density actually decreases in income for the income range in which individuals purchase a positive amount of effort.*

Result 10. *Security is a superior good, i.e. the proportion of income spent on security increases with income. The demand for security obeys the law of demand, i.e. demand decreases in ρ. Supply increases in ρ.*

We omit the proof of Result 10, as it is obvious.

Result 11. *Individuals with zero demand for effort never benefit from private security. If $\xi(q) = 0$ and $S_q > 0$, they will suffer from a higher crime density than without private security.*

Proof This is a variant of Result 7. ■

Result 12. *An increase in the income of individuals in some non-empty interval $[q_3, 1] \subset Q$ can increase the crime density suffered by individuals in some non empty interval $[q_1, q_2] \subset [0, q_3)$, with $q_2 < q_3$.*

Proof Consider Figure 3. The first part shows the old and new income distribution with three distinct income types (solid lines), which we will refer to as "the poor", "the middle class", and "the rich". The income of the poor is below the threshold Y_m, i.e. they will not experience harm from crime. Income is then shifted from the poor to the rich, preserving the mean income. Let $Y_{q,2} = Y_{q,1} + \delta$ be the new income of rich types q. Suppose that the marginal utility from income for the rich is already very low. The increase in income will then lead to an increase in expenditure on effort against crime very close to δ, because (i) the marginal reduction in harm from crime will increase and (ii) the marginal utility from income will decrease (see Figure 3, lower right). This increase in demand will lead to an increase in the price for security (by Result 10), which can lead the middle class to demand less security (see Figure 3, lower left). ■

We summarize the main results of this section in the following (informal) proposition:

Proposition 4. *There exists a competitive crime-security-equilibrium; in any such equilibrium, the equilibrium effort allocation is symmetric. Demand for effort against crime increases in income, effort against crime is a superior good, and it obeys the law of demand. If marginal utility of income is constant, private security equalizes the crime density among those who demand security in equilibrium. If marginal utility of income is decreasing, crime density increases with income up to the marginal income type who demand zero effort, and then decreases in income. Those who do not demand security in equilibrium will not benefit from the effort against crime demanded by others, and will often suffer from a higher crime density. Increasing inequality can shift crime away from those with increased income to those whose income remains constant.*

3.5 *An Example*

It is helpful to continue the example given in Section 2.6. In addition to the previous assumptions, we assume a very convenient form of transformation of

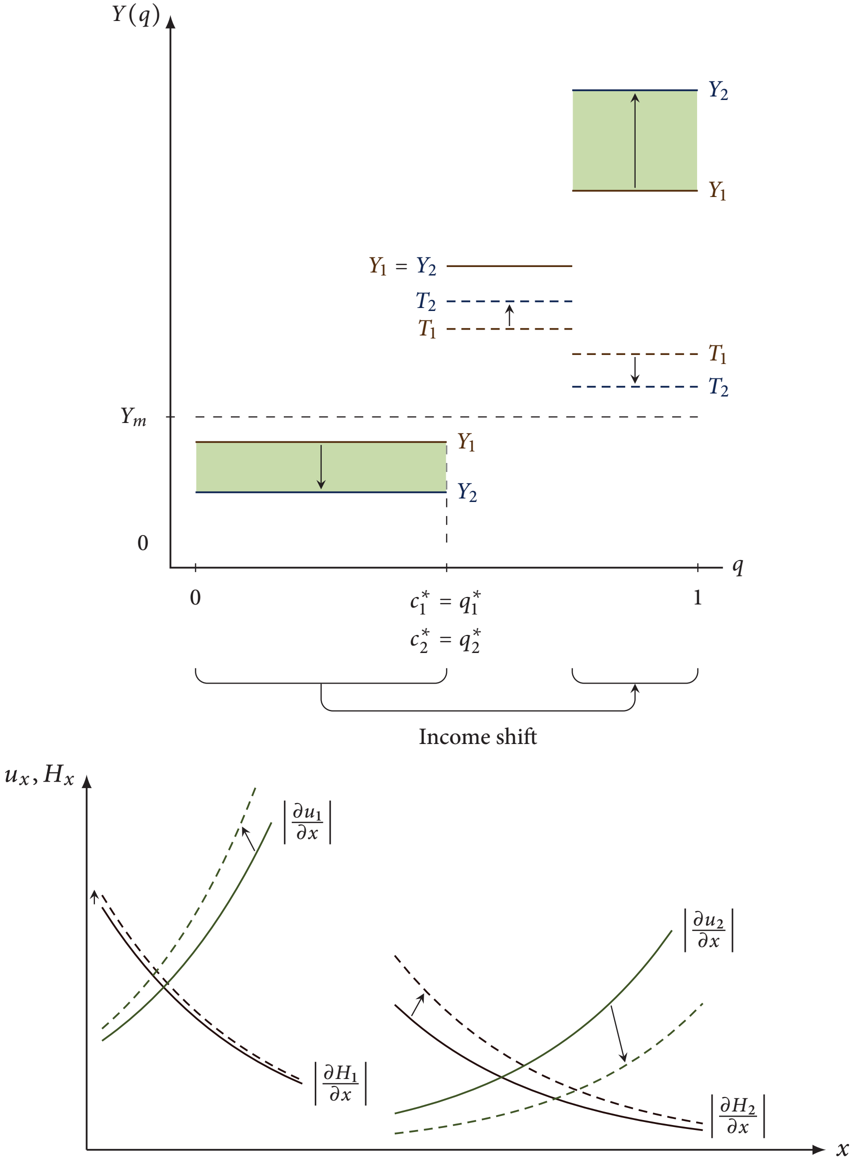


Figure 3: A higher inequality can lead to less crime but a higher crime density for the middle class.

stealable income which satisfies Condition 5: $T(y,x) = \exp(-x/2)\,y$. Then,

$$\begin{aligned}\bar{S} &= (q^*)^{-1} \int_{a \in A_C} \beta\, T(Y_a, \xi[a])\, \mathrm{d}\, a \\ &= (q^*)^{-1} \int_{a \in A_C} \beta\, \exp(-\xi[a]/2)\; Y_a\, \mathrm{d}\, a. \end{aligned} \tag{22}$$

Using Result 9, we know that $T(Y_a, \xi[a])$ must be the same for all players with positive demand for effort against crime. Let us denote the strategy set which consists of player types with positive demand as $(a_0, 1]$. For all $a \in (a_0, 1]$, we must have in equilibrium

$$S^0 = \beta\, \exp(-\tilde{\xi}[a]/2)\; Y_a\, \mathrm{d}\, a \tag{23}$$

for some $S^0 > 0$, where $\tilde{\xi}(a)$ may be negative. Solving Eq. (23) for $\tilde{\xi}(a)$ gives

$$\tilde{\xi}(a) = -2\, \log\left(\frac{S^0}{\beta\, Y_a}\right). \tag{24}$$

Setting $\tilde{\xi}(a) = 0$ and solving for a gives a_0, depending on S^0. In equilibrium, the aggregate effort allocated to the players (i.e., total demand) must equal total supply of effort:

$$\int_{a \in (a_0, 1]} \tilde{\xi}(a)\, \mathrm{d}\, a = e_\Sigma. \tag{25}$$

Solving Eq (25) for S^0 gives the explicit solution

$$S^0 = \beta\, [\exp(\alpha\, e_\Sigma/2) - 1]^{-\frac{1}{a}} \tag{26}$$

With S^0 given, we can then compute the equilibrium effort allocation as

$$\xi(q) = \max\left\{0, \left[\frac{2}{\alpha}\, \log\left(\frac{q}{1-q}\, [\exp(\alpha\, e_\Sigma/2) - 1]\right)\right]\right\} \tag{27}$$

Figure 4 shows the equilibrium effort allocation for different parameters of the income distribution.

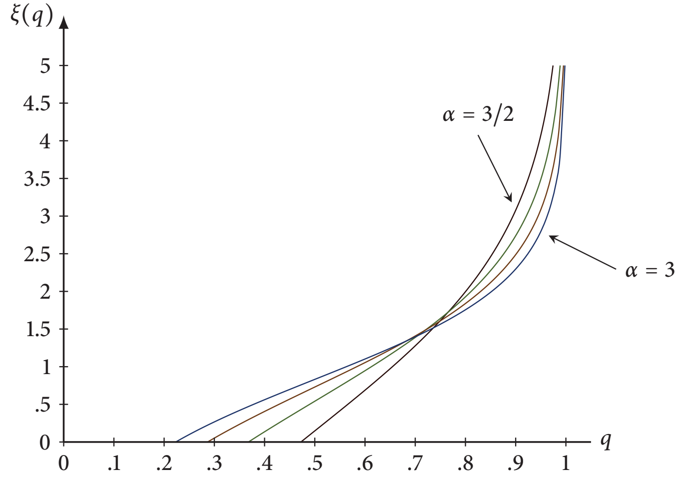


Figure 4: The equilibrium effort allocation for $\alpha \in \{3/2, 2, 5/2, 3\}$, $\beta = 1/10$, $p = 1/2$, and $e_\Sigma = 1$.

The marginal criminal q^* is indifferent between choosing crime and no crime; thus, q^* is the q which solves

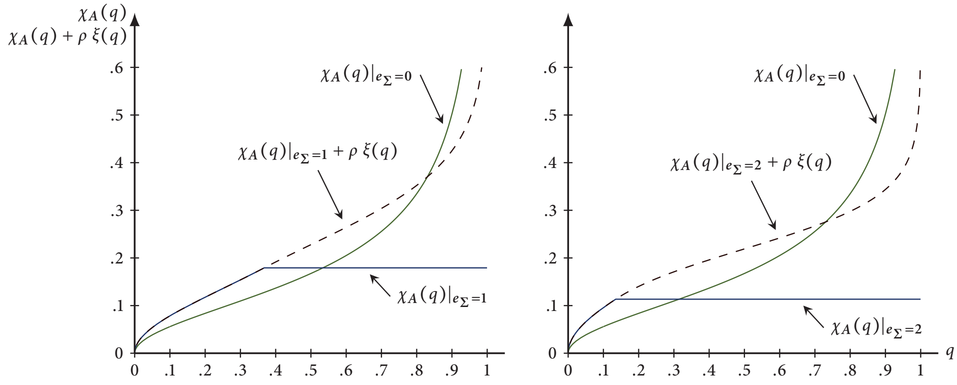
$$(1-p)\left(Y_q + q^{-1} \int_{a\in A_C} \beta \exp(-\tilde{\xi}[a]/2)\, Y_a \,\mathrm{d}\, a\right) = Y_q. \tag{28}$$

With

$$\int_{a\in A_C} \exp(-\tilde{\xi}[a]/2)\, Y_a \,\mathrm{d}\, a = B\left(\exp\left[-\frac{\alpha\, e_\Sigma}{2}\right]; \frac{a+1}{a}, -\frac{1-a}{a}\right) \\ + \left[\exp\left(-\frac{\alpha\, e_\Sigma}{2}\right)\right]\left[\exp\left(\frac{\alpha\, e_\Sigma}{2}\right) - 1\right]^{-\frac{1-a}{a}} \tag{29}$$

we can then find, for rational values of α, explicit (though very lengthy) solutions for the equilibrium crime level, the equilibrium crime distribution, and the market price for any given supply of effort e_Σ (i.e. the inverse demand function).

Figures 5 and 6 show the equilibrium crime distribution for $\alpha = 2$ and $\alpha = 3$, respectively, and two different values of e_Σ. The figures also show the sum of the harm suffered due to crime density and the expenditure for effort against crime (i.e. the market price ρ time the effort demanded). In this particular example, we can simply add up these two components of the utility function because both utility from income and harm from crime density are identity functions. Figure 7 shows the market price or inverse demand function for effort against crime for $\alpha = 2$ and $\alpha = 3$.



(a) $e_\Sigma = 1$ (b) $e_\Sigma = 2$

Figure 5: The equilibrium crime density with and without private security, and the sum of harm and expenditure on effort against crime for $\alpha = 2$, $\beta = 1/10$, and $p = 1/2$.

Consider for example Figure 5.(a). It shows that compared to a situation without private security, the poorest 53% of players will suffer from a higher crime density once private security is introduced. Perhaps even more interestingly, only the richest 17% actually benefit from private security: Players of type $q \in [.53, .83]$ do enjoy a lower crime density, but the expenses for private security more than compensate the positive effect. Players of a type

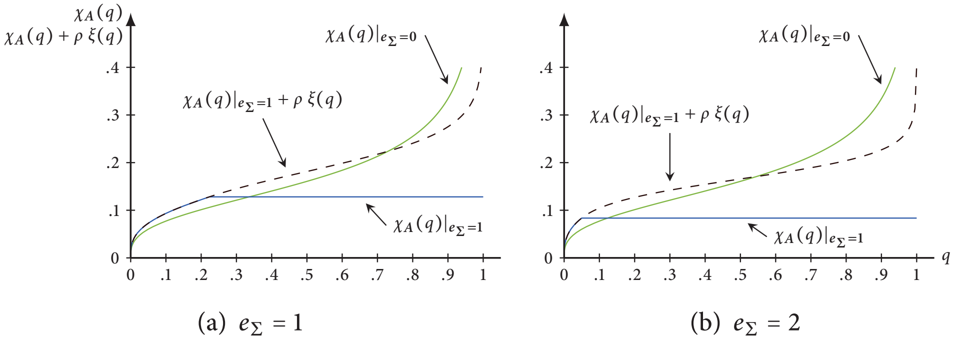


Figure 6: The equilibrium crime density with and without private security, and the sum of harm and expenditure on effort against crime for $\alpha = 3$, $\beta = 1/10$, and $p = 1/2$.

$q \in [.37, .53]$ do not only suffer from a higher crime density than without private security, they even have positive expenditure on private security.

Comparing Figure 5.(a) with 5.(b), we see that an increase in supply of effort against crime will reduce the measure of players who are worse off due to private security; however, those who still do not benefit from private security are even worse off. Comparing Figures 5 and6 shows that as inequality decreases (in the sense of the Lorenz ordering), both negative and positive effects of private security are still present but less pronounced.

3.6 *Discussion*

The competition among PSCs could also, and perhaps more realistically, be modelled as Bertrand-competition with capacity choice, which also leads to Cournot-outcomes (Kreps and Scheinkman 1983).

We have not explicitly modelled emigration. Barenboim and Campante (2008) find that increasing crime levels can lead to the emigration of the middle class, leaving behind only the very poor and the very rich, thus increasing inequality; the rich stay because they can afford privately supplied security. Result 12 suggests that an extension of our model can generate this result as well: An increase in income of high income types can increase the crime

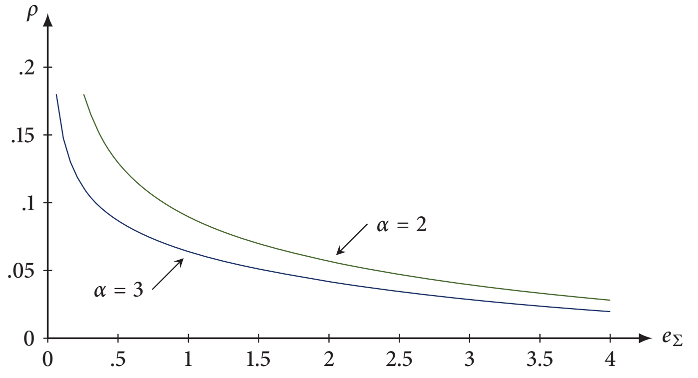


Figure 7: The equilibrium market price for effort (i.e. the inverse demand function), for $\alpha \in \{2, 3\}$, $\beta = 1/10$, and $p = 1/2$.

density at intermediate income types, leaving them worse off. If this effect is strong enough, higher inequality can push utility of the middle class below a threshold level which triggers emigration. News reports and research[4] suggest that this is indeed an effect of the high crime rates in South Africa, leading skilled middle member of the middle class to leave the country.

4 DISCUSSION

4.1 *Possible Extensions*

Besides explicitly modelling of emigration (see Section 3.6), another interesting extension which would introduce dynamics to game would be to consider social mobility. If social mobility is generally high, i.e. the poor can expect to have higher legal incomes in the near future, this might substantially reduce crime, as the expected opportunity costs would increase.

A further generalization of the model would consider individuals being endowed with a time budget which they can spend on normal work (generating

[4] See, for instance, BBC News World Edition, July 24th, 2002: "South Africa Hit by 'Brain-Drain'", http://news.bbc.co.uk/2/hi/africa/2149297.stm; The New York Times, November 1st, 2008: "South African Emigration Leads to Skills Shortage"; Dodson (2002); Bohlman (2010).

the legal income of the model considered here), crime (generating the payoff from crime), and effort against crime which can be sold on the market for a market price ρ. In that case, the increase in the market price caused by increased demand would also reduce crime by increasing opportunity costs: Both security services and crime is "provided" by low income types; higher income from security work would decrease crime.

The increase in the market price for private security can also affect the expenditure and recruitment of the state police. Goodenough (2007) reports that the private security officers in South Africa are often paid more than the regular police. Thus, a higher demand for private security can also lead to a lower level of publicly provided security.

4.2 *Conclusion*

We modelled crime as non-atomic game with decreasing returns to crime as more criminals exploit stealable income. The specification of the model allowed to derive a crime distribution, i.e. a distribution of criminal activity among individuals of different incomes. We showed that there exists an equilibrium crime distribution which equalizes payoff of stealing from any income type for which stealing is profitable at all. There exists a marginal criminal: All individuals with lower income supplement their income with crime, while all individuals with higher income do not commit crimes. Inequality tends to increase crime by (i) lowering the income of poorer individuals and/or (ii) increasing "stealable income"; however, for (i) it matters whether the income of the marginal criminal is among the incomes which decrease; it is possible to find examples for which higher inequality actually leads to less crime. Higher income of high income types can decrease the harm from crime suffered by poor and intermediate income types: Even though crime increases, the crime targeted at high income types increases by more than overall crime, thus shifting some of the crime from poor and intermediate types to the rich.

Introducing private security can substantially change some of the results of the model without private precautions against crime. We showed that there exists a competitive crime-security-equilibrium; in any such equilibrium, the equilibrium effort allocation is symmetric. Demand for effort against crime increases in income, effort against crime is a superior good, and it obeys the law of demand. If marginal utility of income is linear, private security equalizes the crime density among those who demand security in equilibrium. If marginal utility of income is decreasing, crime density increases with income up to the marginal income type who demand zero effort, and then decreases in income. Those who do not demand security in equilibrium will not benefit from the effort against crime demanded by others, and will often suffer from a higher crime density. Increasing inequality can shift crime away from those with increased income to those whose income remains constant.

REFERENCES

Barenboim, I. and F. Campante (2008): Does Crime Breed Inequality? Evidence from the Favelas in Rio de Janeiro, working Paper.

Becker, G. S. (1968): Crime and Punishment: An Economic Approach, *Journal of Political Economy*, 76(2), 169–217.

Becker, G. S. and G. J. Stigler (1974): Law Enforcement, Malfeasance, and Compensation of Enforcers, *Journal of Legal Studies*, 3(1), 1–18.

Ben-Shahar, O. and A. Harel (1995): Blaming the Victim: Optimal Incentives for Private Precautions against Crime, *Journal of Law, Economics, and Organization*, 11(2), 434–455.

Bernasco, W. and P. Nieuwbeerta (2005): How do Residential Burglars Select Target Areas? A New Approach to the Analysis of Criminal Location Choice, *British Journal of Criminology*, 44(3), 296–315.

Block, M. K. and J. M. Heineke (1975): A Labor Theoretic Analysis of the Criminal Choice, *American Economic Review*, 65(3), 314–325.

Bohlman, H. R. (2010): The Macroeconomic Impact of Skilled Emigration from South Africa: A CGE Analysis, centre of Policy Studies, Monash University, Working Paper Number 166.

Choe, J. (2008): Income Inequality and Crime in the United States, *Economics Letters*, 101(1), 31–33.

Clotfelter, C. T. (1978): Private Security and the Public Safety, *Journal of Urban Economics*, 5(3), 388–402.

Collett-Schmitt, K. E. (2007): Observable Private Precaution and Its Effect on Crime: The Case of Burglar Alarms, north Carolina State University, Working Paper.

Corman, H. and H. N. Mocan (2000): A Time-Series Analysis of Crime, Deterrence, and Drug Abuse in New York City, *American Economic Review*, 90(3), 584–604.

Demombynes, G. and O. Berk (2005): Crime and Local Inequality in South Africa, *Journal of Development Economics*, 76(2), 265–292.

Di Tella, R., S. Galiani, and E. Schargrodsky (2002): Crime Victimization and Income Distribution, inter-American Development Bank.

Di Tella, R. and E. Schargrodsky (2004): Do Police Reduce Crime? Estimates Using the Allocation of Police Forces After a Terrorist Attack, *American Economic Review*, 94(1), 115–133.

Dodson, B. (2002): Gender and the Brain Drain from South Africa, the Southern African Migration Project. Migration Policy Series No. 23.

Ehrlich, I. (1973): Participation in Illegitimate Activities: A Theoretical and Empirical Investigation, *Journal of Political Economy*, 81(3), 521–565.

——— (1975): The Deterrent Effect of Capital Punishment: A Question of Life and Death, *American Economic Review*, 65(3), 397–417.

——— (1996): Crime, Punishments, and the Market for Offenses, *Journal of Economic Perspectives*, 10(1), 43–67.

Evans, D. J. (1989): Geographical Analysis of Residential Burglary, in: D. J. Evans and D. T. Herbert (Editors), *The Geography of Crime*, London: Routledge, 86–107.

Friedman, D. (1984): Efficient Institutions for the Private Enforcement of Law, *Journal of Legal Studies*, 13(2), 379–397.

Gastwirth, J. L. (1971): A General Definition of the Lorenz Curve, *Econometrica*, 39(6), 1037–1039.

Gonzalez-Navarro, M. (2008): Deterrence and Displacement in Auto Theft, cEPS Working Paper No. 177.

Goodenough, C. (2007): Joining Forces Against Crime. Potential for the Private Security Industry to Become Proactive, *SA Crime Quarterly*, 20, 7–12.

Helsley, R. W. and W. C. Strange (1999): Gated Communities and the Economic Geography of Crime, *Journal of Urban Economics*, 46(1), 80–105.

Irish, J. (1999): *Policing for Profit: The Future of South Africa's Private Security Industry*, volume 39 of *ISS Monograph Series*, Institute for Security Studies.

Kelly, M. (2000): Inequality and Crime, *Review of Economics and Statistics*, 82(4), 530–539.

Kreps, D. M. and J. A. Scheinkman (1983): Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, *Bell Journal of Economics*, 14(2), 326–337.

Landes, W. M. and R. A. Posner (1975): The Private Enforcement of Law, *Journal of Legal Studies*, 4(1), 1–46.

Mas-Colell, A. (1984): On a Theorem of Schmeidler, *Journal of Mathematical Economics*, 13(3), 201–206.

Neumayer, E. (2005): Inequality and Violent Crime: Evidence from Data on Robbery and Violent Theft, *Journal of Peace Research*, 42(1), 101–112.

Polinsky, A. M. (1980): Private versus Publich Enforcement of Fines, *Journal of Legal Studies*, 9(1), 105–127.

Prinsloo, J. and C. Marais (2006): The Multilateralisation of Policing in South Africa, *Strategic Review for Southern Africa*, 73–88.

Private Security Industry Regulatory Authority (2007): Annual Report 2006/2007.

——— (2009): Annual Report 2008/2009.

Rath, K. P. (1992): A Direct Proof of the Existence of Pure Strategy Equilibria in Games with a Continuum of Players, *Economic Theory*, 2(3), 427–433.

Rath, K. P., Y. Sun, and S. Yamashige (1995): The Nonexistence of Symmetric Equilibria in Anonymous Games with Compact Action Spaces, *Journal of Mathematical Economics*, 24(4), 331–346.

Roughgarden, T. and E. Tardos (2004): Bounding the Inefficiency of Equilibria in Nonatomic Congestion Games, *Games and Economic Behavior*, 47(2), 389–403.

Schmeidler, D. (1973): Equilibrium Points of Nonatomic Games, *Journal of Statistical Physics*, 7(4), 295–300.

Shavell, S. (1991): Individual Precautions to Prevent Theft: Private Versus Socially Optimal Behavior, *International Review of Law and Economics*, 11(2), 123–132.

Singh, S. and G. S. Maddala (1976): A Function for Size Distribution of Incomes, *Econometrica*, 44(5), 963–970.

Wilfling, B. and W. Krämer (1993): The Lorenz-Ordering of Singh-Maddala Income Distributions, *Economics Letters*, 43(1), 53–57.

Witt, R., A. Clarke, and N. Fielding (1999): Crime and Economic Activity: A Panel Data Approach, *British Journal of Criminology*, 39(3), 391–400.

Witte, A. D. (1980): Estimating the Economic Model of Crime with Individual Data, *Quarterly Journal of Economics*, 94(1), 57–84.