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A Survey of Elasticities

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Manuel Frondel[1]

Modeling Energy and Non-energy Substitution – A Survey of Elasticities

Abstract

Estimating the degree of substitution between energy and non-energy inputs is key for any evaluation of environmental and energy policies. Yet, given the large variety of substitution elasticities, the central question arises as to which measure would be most appropriate. Apparently, ALLEN's elasticities of substitution have been the most-used measures in applied production analysis. In line with Frondel (2004), this paper argues that cross-price elasticities are preferable for many practical purposes. This conclusion is based on a survey of classical substitution measures, such as those from ALLEN, MORISHIMA, and MCFADDEN. The survey also highlights the fact that cross-price elasticities are their essential ingredients.

JEL Classification: C3, D2

Keywords: Cross-price elasticities; Allen partial elasticities; Morishima elasticities

May 2011

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1 Introduction

Estimating the degree of substitution between production factors such as energy and non-energy inputs is crucial for a host of issues, including environmental and energy policies such as trading greenhouse gas emission allowances, recycling energy tax revenues to reduce output or non-energy factor taxes, and the step-by-step increase of fuel taxes. Another example is the impact of fuel efficiency gains on energy use, which is also largely driven by the ease of factor substitution (SAUNDERS 2008, 1992).

Yet, despite the fact that a large number of empirical studies have appeared since the first energy crisis in the 1970's, there seems to be little consensus on the degree and even the direction of energy substitution. For instance, ever since BERNDT and WOOD's (1975) finding that the energy aggregate complements capital, and GRIFFIN and GREGORY's (1976) results indicating that both factors are substitutes, the energy-capital debate has remained unresolved – for surveys, see KINTIS and PANAS (1989), APOSTOLAKIS (1990), and FRONDEL and SCHMIDT (2002, 2003).

Although there are other important causes of divergent results, such as the industries and regions under study, this paper focuses on one important source: the large variety of distinct measures of substitution. Since HICKS (1932) originally defined the unique substitution measure σ for the case of only two inputs, often called *the* elasticity of substitution, many different generalizations of this fundamental concept up to an arbitrary number of inputs have been provided – see ALLEN and HICKS (1934), ALLEN (1938), UZAWA (1962), MCFADDEN (1963), MORISHIMA (1967), BLACKORBY and RUSSELL (1989). Facing such a variety of measures and given the variation in perspectives and interpretations among substitution elasticities, the central question arises as to which substitution measure would be most appropriate in an empirical study.

Apparently, ALLEN's partial elasticities of substitution (AES) have played a dominant role and have been the most-used measures of substitution in the production literature – see e. g. HAMERMESH (1993:35). AES, however, has been criticized in the literature as only being interpretable in terms of cross-price elasticities. AES is thus argued to add no more information to that already contained in cross-price elasticities (BLACKORBY and RUSSELL 1989:883).

Along the lines of FRONDEL (2004) and FRONDEL and SCHMIDT (2006), who focus on the classical cross-price elasticities when measuring the ease of substitution among energy and non-energy inputs, this paper argues that analysts are frequently better served by appealing to cross-price elasticities. This argument is supported here by a survey of classical substitution measures including ALLENS's partial elasticities of substitution (AES), MORISHIMA's partial elasticities of substitution (MES), and MCFADDEN's shadow elasticities of substitution (SES). The survey illustrates that all these standard measures are founded on cross-price elasticities.

This article's main contribution relies on demonstrating that analysts must take great care in interpreting the standard substitution elasticities commonly employed. Whenever one draws conclusions from empirical studies on the degree and direction of substitutability of production factors, it is indispensable to, first, clearly indicate the particular measure employed to denote two inputs as substitutes and, second, to interpret empirical results accordingly in order to avoid harmful policy recommendations. Ultimately, it becomes obvious that there cannot be a universally applicable substitution elasticity. Instead, the selection of a particular measure critically depends on the concrete application and question asked, a conclusion that can be traced to MUNDLAK (1968:234).

2 A Survey of Classical Substitution Elasticities

When discussing elasticities of substitution, it is convenient and intuitive to commence with *the* elasticity of substitution, originally introduced by HICKS (1932) for the analysis of only two factors. HICKS's σ measures the relative change in the factor proportion x_1/x_2 due to the relative change in the marginal rate of technical substitution f_{x_2}/f_{x_1} while output Y is held constant:

$$\sigma = \frac{d \ln \left(\frac{x_1}{x_2} \right)}{d \ln \left(\frac{f_{x_2}}{f_{x_1}} \right)}. \tag{1}$$

With more than two factors being flexible, the marginal rate of technical substitution f_{x_2}/f_{x_1} would not be determined uniquely. To avoid such ambiguities in a multi-factor setting, further assumptions that are discussed below are necessary.

An alternative definition of σ in the two-dimensional case, which BLACKORBY and RUSSELL (1989) call the HICKS' elasticity of substitution (HES), can be obtained under the assumptions of perfect competition and profit maximization, so that f_{x_2}/f_{x_1} equals relative factor prices p_2/p_1:

$$\text{HES} = \frac{d \ln \left(\frac{x_1}{x_2} \right)}{d \ln \left(\frac{p_2}{p_1} \right)}. \tag{2}$$

It is this definition (2) that serves as a basis for all generalizations of σ for a multi-factor setting. Since output is assumed to be constant, the following generalizations inherit this property.

The literature's consensus of an ideal concept of multi-factor substitution is to report optimal adjustment in relative inputs x_i/x_j when merely the relative input price of two arbitrary factors i and j changes, with all inputs being flexible and cost minimized for fixed output.[1] This measure is often called HICKS-ALLEN

[1]The most general measure of substitution on the basis of (2) would be a concept of *total sub-*

elasticity of substitution (HAES), where

$$\text{HAES}_{ij} = \frac{\partial \ln\left(\dfrac{x_i}{x_j}\right)}{\partial \ln\left(\dfrac{p_j}{p_i}\right)} = \frac{\partial \ln x_i}{\partial \ln\left(\dfrac{p_j}{p_i}\right)} - \frac{\partial \ln x_j}{\partial \ln\left(\dfrac{p_j}{p_i}\right)}, \tag{3}$$

and only the relative price of two factors i and j changes. If apart from i and j all other factors are assumed to be constant, HAES_{ij} is in fact HICKS' elasticity of substitution HES.

While HAES_{ij} measures the relative change of the input proportion x_i/x_j, and therefore may be termed a measure of relative substitutability, the cross-price elasticity

$$\eta_{x_i p_j} := \frac{\partial \ln x_i}{\partial \ln p_j} \tag{4}$$

may be termed a measure of absolute substitutability, because it focuses on the relative change of a single factor i due to a sole change of the price of factor j, with output and all other prices being fixed. Thus, according to MUNDLAK's (1968) classification, $\eta_{x_i p_j}$ is a *one-price-one-factor* elasticity of substitution.

It is now shown that cross-price elasticities are the common basis of AES, MES, and SES. First, AES is related to $\eta_{x_i p_j}$ by

$$\text{AES}_{ij} = \frac{\eta_{x_i p_j}}{s_j}, \tag{5}$$

where $s_j = \frac{x_j p_j}{C}$ denotes the cost share of factor j – see e. g. FRONDEL and SCHMIDT (2004:220). While expression (5) reveals that the truly interesting measure is $\eta_{x_i p_j}$, AES has been the most extensively used elasticity of substitution in empirical studies (HAMERMESH, 1993:35).

Second, MES is most generally defined by

$$\text{MES}_{x_i p_j} := \frac{\partial \ln(x_i/x_j)}{\partial \ln p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} = \eta_{x_i p_j} - \eta_{x_j p_j} \tag{6}$$

stitution, where besides p_i and p_j all other prices are flexible as well. According to MUNDLAK (1996:232), however, such "a concept [...] may have little to contribute".

and is a *two-factor-one-price* elasticity, where solely the price of factor j is flexible, again with all other prices being fixed (BLACKORBY and RUSSELL, 1989). Similar to cross-price elasticities, but unlike AES, MES is asymmetric: In general, $\text{MES}_{x_i p_j} \neq \text{MES}_{x_j p_i}$. It becomes transparent from definition (6) that if one were to classify two factors using MES, one would more frequently conclude that these factors are substitutes than if one were using AES or cross-price elasticities. The reason is that even if $\eta_{x_i p_j}$ is negative and thus factor i and j are termed complements, $\text{MES}_{x_i p_j}$ may be positive, hence indicating substitutability, if the magnitude of the always negative own-price elasticity is sufficiently large.

We argue that for many practical purposes, cross-price elasticities should be favored over MES. The reason is that it is frequently more interesting to get to know how the use of factor i is changing due to an exogenous increase in the price p_j of factor j, rather than to learn something about the change of the input proportion x_i/x_j, as would be measured by $\text{MES}_{x_i p_j}$. If, for instance, oil prices are soaring, politicians would rather want to know how much of a detrimental impact the high prices will have on the labor input of the economy alone than to know how the labor-energy input proportion changes and whether the use of either labor or energy is more reduced due to surging oil prices. Hence, notwithstanding the significance of MES as the sole true generalization of HICKS' σ (BLACKORBY and RUSSELL, 1989), estimating cross-price elasticities, rather than any substitution measure involving input ratios, frequently appears to be more appropriate in empirical studies on issues such as the consequences of energy price policies.

Third, the *two-factor-two-price* elasticity HAES_{ij} is a weighted average of $\text{MES}_{x_i p_j}$ and $\text{MES}_{x_j p_i}$.

Proof: Given $x_i = x_i(p_1, ..., p_n)$ and using the chain rule, we have

$$\frac{\partial \ln x_i}{\partial \ln(\frac{p_j}{p_i})} = \frac{\partial \ln x_i}{\partial \ln p_i} \cdot \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + \frac{\partial \ln x_i}{\partial \ln p_j} \cdot \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} = \eta_{x_i p_i} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + \eta_{x_i p_j} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}, \tag{7}$$

because merely the prices p_i and p_j are flexible. Therefore, we also get

$$\frac{\partial \ln x_j}{\partial \ln(\frac{p_j}{p_i})} = \eta_{x_j p_i} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + \eta_{x_j p_j} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}. \tag{8}$$

Altogether, we obtain

$$\begin{aligned} \mathrm{HAES}_{ij} &= \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln(\frac{p_j}{p_i})} = \frac{\partial \ln x_i}{\partial \ln(\frac{p_j}{p_i})} - \frac{\partial \ln x_j}{\partial \ln(\frac{p_j}{p_i})} \\ &= \underbrace{(\eta_{x_i p_j} - \eta_{x_j p_j})}_{\mathrm{MES}_{x_i p_j}} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} - \underbrace{(\eta_{x_j p_i} - \eta_{x_i p_i})}_{\mathrm{MES}_{x_j p_i}} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})}, \end{aligned} \tag{9}$$

where the weights add to unity:

$$\frac{\partial \ln p_j}{\partial \ln(p_j/p_i)} + (-\frac{\partial \ln p_i}{\partial \ln(p_j/p_i)}) = \frac{\partial \ln(p_j/p_i)}{\partial \ln(p_j/p_i)} = 1. \tag{10}$$

The weighted sum given in (9) reflects the fact that there is an infinite number of changes of prices p_i and p_j that lead to the same change of price ratio p_j/p_i. There are two polar cases: If only p_j changes and p_i is fixed, HAES_{ij} equals $\mathrm{MES}_{x_i p_j}$, while, vice versa, HAES_{ij} specializes to $\mathrm{MES}_{x_j p_i}$ if only p_i changes and p_j is fixed.

To complete the survey, it is proved that MCFADDEN's *shadow elasticity of substitution* (SES), which is a special case of the definition (3) underlying HAES in that in contrast to HAES it holds cost constant, is also a weighted average of $\mathrm{MES}_{x_i p_j}$ and $\mathrm{MES}_{x_j p_i}$.

Proof: While SES fixes cost $C = C(p_1, ..., p_n)$ and only two prices p_i and p_j are supposed to change, on the basis of SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i} = x_i$, and the chain rule follows:

$$0 = \frac{\partial C}{\partial(\frac{p_j}{p_i})} = \frac{\partial C}{\partial p_i} \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + \frac{\partial C}{\partial p_j} \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})} = x_i \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + x_j \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})}. \tag{11}$$

By dividing (11) by C, one obtains

$$0 = (\frac{1}{p_i}) \cdot (\frac{p_i x_i}{C}) \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + (\frac{1}{p_j}) \cdot (\frac{p_j x_j}{C}) \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})} = s_i \cdot \frac{\partial \ln p_i}{\partial(\frac{p_j}{p_i})} + s_j \cdot \frac{\partial \ln p_j}{\partial(\frac{p_j}{p_i})}. \tag{12}$$

Multiplying by p_j/p_i leads to

$$0 = (\frac{p_j}{p_i}) \cdot s_i \cdot \frac{\partial \ln p_i}{\partial(\frac{p_j}{p_i})} + (\frac{p_j}{p_i}) \cdot s_j \cdot \frac{\partial \ln p_j}{\partial(\frac{p_j}{p_i})} = s_i \cdot \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + s_j \cdot \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}. \tag{13}$$

Combining equation (10) and the right-hand side of equation (13) yields

$$\frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} = -\frac{s_j}{s_i + s_j} \quad \text{and} \quad \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} = \frac{s_i}{s_i + s_j}. \tag{14}$$

By plugging both derivatives into the right-hand side of (9), we finally get

$$\text{SES}_{ij} = (\frac{s_i}{s_i + s_j})\text{MES}_{x_i p_j} + (\frac{s_j}{s_i + s_j})\text{MES}_{x_j p_i}. \tag{15}$$

Note that the symmetry of this expression indicates that, unlike MES, SES is symmetric.

In sum, two common features of AES, MES, HAES, and SES become apparent in this section. First, all these elasticities ignore output effects and, second, all are mixtures of cross-price elasticities. While HAES is the most general of the presented measures, because it captures factor substitution when two factor prices are flexible, this generality is also the reason for HAES being of minor practical importance: It is simply not possible to obtain from HAES a single substitution estimate for any two factors without specifying how these two factor prices change.

By contrast, apart from SES, which also measures substitution relationships when two prices are flexible, yet under the additional, restrictive assumption that cost are constant, not just output, all other measures described in this section are based on the assumption that only one factor price alters. It could be argued, however, that in modeling practise one is frequently confronted with counterfactual situations describing what would happen if the price of only a single factor were to drastically increase. In modeling industrial energy consumption, for instance, this is a rather typical situation, as, most importantly, oil prices are highly volatile and are frequently doubling within short periods of time.

In any case, this didactic survey should have demonstrated that whenever one draws conclusions from empirical studies on the degree of substitutability of two inputs, it is indispensable to, first, clearly indicate the particular measure employed to denote these inputs as substitutes and, second, to interpret empirical results accordingly.

3 Summary and Conclusion

Given the multitude of generalizations of HICKS' σ, the unique elasticity of substitution for the two-factor case, the central question arises as to which measure would be appropriate to capture substitution relationships such as those between energy and non-energy inputs. In a multi-factor setting, ALLEN's elasticities of substitution (AES) apparently have been the most-used measures in applied production analysis. BLACKORBY and RUSSELL (1989:883), however, criticize that AES adds no more information to that already contained in cross-price elasticities.

On the basis of a survey of σ's most prominent generalizations, including AES, HICKS-ALLEN's (HAES), MORISHIMA's (MES), and MCFADDEN's shadow elasticities of substitution (SES), this paper has shown that cross-price elasticities play a fundamental role in measuring substitution issues, since they are the common basis for AES, MES, and SES. Moreover, it is argued that cross-price elasticities are often more relevant in terms of economic content. The ultimate reason for this conclusion is that cross-price elasticities measure the relative change of only one factor due to price changes of another input, whereas HAES, MES, and SES measure the relative change of a *factor ratio* due to price changes of these two factors.

While measuring the relative change of a factor ratio appears to be of minor importance for many applications, we argue that any substitution measure has to match the specific task it is employed for and emphasize FUSS, MCFADDEN and MUNDLAK's (1978:241) conclusion that there "is no unique natural generalization of the two factor definition ... [and] the selection of a particular definition should depend on the question asked". Hence, a clear understanding of the differences in interpretations and perspectives captured by the variety of substitution measures is indispensable.

Yet, all the presented elasticities solely measure pure substitution effects; that is, they ignore output effects, because constancy of output is the maintained hypothesis underlying these concepts. Oil price shocks, however, indicate that it is frequently problematic to ignore output effects in empirical studies of factor substitution. As it is most likely that output shrinks when the price of a factor such as energy rises, elasticities capturing gross substitution effects – that is, pure substitution and output effects – are preferable in any empirical study. Based on the argument that cross-price elasticities are often more relevant for many practical purposes, a generalization of cross-price elasticities that allows for output variations would be a possible candidate concept.

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