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Empirical Evidence from the United States

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Mario Jovanović[1]

Does Monetary Policy Affect Stock Market Uncertainty? – Empirical Evidence from the United States

Abstract

This paper investigates the response of US stock market uncertainty to monetary policy of the Federal Reserve Bank. It can be shown that monetary policy significantly Granger-causes stock market confidence. By using monthly closing prices of the V IX as a stock market uncertainty proxy and a copula-based Markov approach the stable nonlinear relation between confidence and uncertainty is demonstrated. The monetary policy effect on stock market uncertainty is therefore separable into a linear and nonlinear part.

JEL Classification: C12, C22, E43, E52

Keywords: Stock market confidence; temporal dependence; copula

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1 Ruhr-Universität Bochum. *– I would like to thank Manfred Lösch and Michael Roos for extensive discussions and Sue Man Fan, Birte Homann, Anna Klabunde, Wolfgang Luhan and Andreas Orland for helpful comments.* *– All correspondence to Mario Jovanović, Ruhr-Universität Bochum, Universitätsstr. 150, 44801 Bochum, Germany, E-Mail: mario.jovanovic@rub.de.*

1 Introduction

Missing confidence was the core problem of the latest financial market crisis and has refuelled the discussion about the behavioral channel of monetary policy. Rajan (2006) argue informally that low interest rate policies conducted by a central bank may shift the investment behavior of investors to risky strategies. This monetary policy transmission can lead to financial instability and can affect stock market uncertainty. Regarding this literature Bekaert and Hoerova (2010) investigate the direct effect of monetary policy on stock market uncertainty, which seems to be insignificant. However, they derive a link between monetary policy and risk aversion, which potentially affects stock market uncertainty. Hence, it is an unsolved question in monetary economics, if monetary policy affects stock market uncertainty. The aim of this paper is to answer this open question and to derive empirical evidence of the existence of a behavioral channel of monetary policy. The existence of the behavioral channel leads to the conclusion that the Federal Reserve Bank is able to tame financial excesses.

In order to investigate the effect of monetary policy on stock market uncertainty the entire effect will be separated into a linear and nonlinear part. It is shown that monetary policy Granger-cause stock market confidence in a linear way. Hence, linear methods are appropriate tools for investigating the impact of macroeconomic indicators on stock market confidence. Contrary, the link between confidence and uncertainty is strongly nonlinear and is introduced on a macroeconomic level by a game with strategic complementarities (see Cooper (1999)).

In the opposite direction stock market uncertainty may also affect monetary policy. Fornari and Mele (2009) show that stock market uncertainty shocks predict economic activity and leads to a sharp drop in employment and output (Bloom (2009)). Therefore, it is conceivable that the monetary authority respond to stock market uncertainty, because it contains information about future economic outcomes. Jovanović and Zimmermann (2010) confirm this conjecture showing that the nominal US interest rate (Federal Funds Rate) significantly reacts to stock market uncertainty in an uncertainty augmented Taylor rule.

The so-called VIX index, which deals with implied volatility, is a popular proxy for financial market uncertainty. The index is designed to measure the market's expectation of 30-day variability implied by at-the-money S&P 500 option prices and is published by the Chicago Board Options Exchange since 1990. Based on an economic model this paper identifies temporal dependence of stock market uncertainty as a proxy for stock market confidence. Bollerslev, Sizova, and Tauchen (2009) investigate the temporal dependence structure of intra-day VIX data for real-time trading. By using low frequency data (monthly) we put the focus on uncertainty trends, which is an issue in economcs.

In order to derive a proxy for stock market confidence, copula-based Markov models are applied as the methodological framework. By the theorem of Sklar (1959) any multivariate distribution can be expressed in terms of its marginal distributions and its copula function. A copula function is a multivariate distribution function with standard uniform marginals, which captures the scale-free dependence structure of the multivariate distribution function. The copula-based approach has the advantage of separating the information about the marginal distributions from the scale-free dependence structure. Darsow, Nguyen, and Olsen (1992) extent this approach to Markov processes. By coupling different marginal distributions with different copula functions, copula-based time series models are able to model a wide variety of marginal behaviors (such as skewness and fat tails) and dependence properties (such as nonlinearities, clusters and tail dependence). Chen and Fan (2006) develop a two-step estimation procedure for parametric copula functions and derived the so called generalized semiparametric regression transformation model. This innovative statistical framework is usable for nonlinear Markov models and augment available linear AR models. The main methodological contribution of this paper is the introduction of copula-based Markov models in economics. Furthermore, it is the time that this recently developed statistical method is used for the deviation of a proxy for stock market confidence. In general, copula-based Markov models augment available econometric tools and can be potentially applied to a wide range of economic questions.

The rest of the paper is organized as follows. Section 2 reviews the methodological concept of copula-based Markov processes and derives a proxy for conditional temporal dependence. Section 3 presents the economic model of stock market confidence. Section 4 outlines the statistical copula specification, whereas Section 5 presents causal effects of macroeconomic indicators for the stock market. Section 6 concludes. Technical details are relegated to the Appendix.

2 Methodology

Let $\{Y_t\}$ be a stationary first-order Markov process with continuous state space. Then the joint distribution function $H(y_{t-1}, y_t) = P(Y_{t-1} \leq y_{t-1}, Y_t \leq y_t)$, $(y_{t-1}, y_t) \in \mathbb{R}^2$, of Y_{t-1} and Y_t completely determines the stochastic properties of $\{Y_t\}$. Due to Sklar's Theorem, it is possible to express $H(y_{t-1}, y_t)$

in terms of the marginal distribution $G(y_t) = P(Y_t \leq y_t)$, $y_t \in \mathbb{R}$, of Y_t and the dependence function of Y_{t-1} and Y_t. This dependence function

$$C(G(y_{t-1}), G(y_t)) = H(y_{t-1}, y_t) \tag{1}$$

is known as "copula". Hence, $C(u_{t-1}, u_t) = P(U_{t-1} \leq u_{t-1}, U_t \leq u_t)$, $(u_{t-1}, u_t) \in [0,1]^2$, is the joint distribution function of the two random variables $U_{t-1} = G(Y_{t-1})$ and $U_t = G(Y_t)$. $h(\cdot,\cdot)$, $c(\cdot,\cdot)$ and $g(\cdot)$ are the associated (joint) density functions. In this paper we will consider three frequently used copulas (Gauss, Clayton, Frank) and one rarely used copula (Fang). For details see the Appendix. One obvious feature of the copula-based time series approach is the possibility to separate the time dependence structure from the marginal distribution. Especially in economics this issue is important, due to the large amount of economic information reflected by the marginal distribution.[1] We make the following set of assumptions:

(A1) $\{Y_t\}_{t=1}^n$ is a sample from a stationary first-order Markov process generated from the true marginal distribution $G(\cdot)$ - which is invariant and absolutely continuous with respect to the Lebesgue measure on the real line - and the true parametric copula $C(\cdot,\cdot;\alpha)$ - which is absolutely continuous with respect to the Lebesgue measure on $[0,1]^2$.
(A2) $G(\cdot)$ and the d-dimensional copula parameter $\alpha \in \mathbb{R}^d$ are unknown.
(A3) $C(\cdot,\cdot;\alpha)$ is neither the Fréchet-Hoeffding upper bound ($C(u_{t-1}, u_t) = \min(u_{t-1}, u_t)$) nor the lower bound ($C(u_{t-1}, u_t) = \max(u_{t-1} + u_t - 1, 0)$).

If (A3) were not true, it is well-known that Y_t would be almost surely a monotonic function of Y_{t-1}. Therefore, the resulting time series would be deterministic and in case of stationarity, $Y_t = Y_{t-1}$ for the upper bound and $Y_t = G^{-1}(1 - G(Y_{t-1}))$ for the lower bound would follow. We abstract from these cases to focus on stochastic samples of stationary first-order Markov processes. Due to Sklar's Theorem of equation (1) the copula density function $c(u_{t-1}, u_t; \alpha) = \frac{\partial^2 C(u_{t-1}, u_t; \alpha)}{\partial u_{t-1} \partial u_t}$ equals $\frac{h(y_{t-1}, y_t)}{g(y_{t-1}) \cdot g(y_t)}$. Hence, the conditional density of y_t given $y_{t-1}, \ldots, y_1$ is

$$h(y_t | y_{t-1}) = g(y_t) c(G(y_{t-1}), G(y_t); \alpha) \ . \tag{2}$$

As far as the conditional density is a function of the copula and the marginal, the v_t-th, $v_t \in [0,1]$, conditional quantile Q_{v_t} of y_t given y_{t-1} is a function of

[1]Furthermore, the temporal dependence structure is invariant concerning monotonic transformations by the invariance theorem of copulas . Hence, temporal dependence of the VIX equals the temporal dependence structure the frequently used transformations VIX^2 and $\ln VIX$.

the copula and the marginal,

$$Q_{v_t}(y_t|y_{t-1}) = G^{-1}\left(C^{-1}_{t|t-1}[v_t|G(y_{t-1});\alpha]\right) \ . \tag{3}$$

$C_{t|t-1}(u_t|u_{t-1};\alpha) = P(U_t \leq u_t|U_{t-1} = u_{t-1}) = \frac{\partial C(u_{t-1},u_t;\alpha)}{\partial u_{t-1}}$ denotes the conditional distribution of U_t given $U_{t-1} = u_{t-1}$, which we assume to exist. Therefore, $C^{-1}_{t|t-1}[v_t|G(y_{t-1});\alpha]$ is the v_t-th conditional quantile of u_t given u_{t-1}. Considering assumption (A2) the unknown marginal distribution $G(\cdot)$ and the unknown copula parameter vector α have to be estimated. Chen and Fan (2006)[2] derive the following semiparametric two-step procedure:

Step 1: Estimate $G(y)$ by the rescaled empirical distribution

$$\widehat{G}(y) = \frac{1}{n+1}\sum_{t=1}^{n} 1\{Y_t \leq y\} \ . \tag{4}$$

Step 2: Estimate the copula parameter vector by

$$\widehat{\alpha} = \arg\max_{\alpha} \frac{1}{n}\sum_{t=2}^{n} \log c(\widehat{G}(Y_{t-1}), \widehat{G}(Y_t);\alpha) \ . \tag{5}$$

$\widehat{\alpha}$ is root-n consistent and has approximately a normal distribution.

According to Chen and Fan (2006) the following generalized semiparametric regression transformation model exists:

$$\Lambda_1(G(Y_t)) = \Lambda_2(G(Y_{t-1})) + \nu_t \ , \ E(\nu_t|Y_{t-1}) = 0 \ , \tag{6}$$

with a parametric increasing function $\Lambda_1(\cdot)$ of U_t, $\Lambda_2(u_{t-1}) := E(\Lambda_1(U_t)|U_{t-1} = u_{t-1})$, and the conditional density of ν_t given $U_{t-1} = u_{t-1}$ is

$$f_{\nu_t|U_{t-1}=u_{t-1}}(\nu_t) = \frac{c(u_{t-1}, \Lambda_1^{-1}(\nu_t + \Lambda_2(u_{t-1}));\alpha)}{\frac{d\Lambda_1(\nu_t+\Lambda_2(u_{t-1}))}{d\nu_t}} \ . \tag{7}$$

It follows in general

$$\Lambda_2(u_{t-1}) = E(\Lambda_1(U_t)|U_{t-1} = u_{t-1}) = \int_0^1 \Lambda_1(u_t)c(u_{t-1}, u_t;\alpha)du_t \tag{8}$$

and for the special case of identity mapping $\Lambda_1(u_t) = u_t$

$$\Lambda_2(u_{t-1}) = E(U_t|U_{t-1} = u_{t-1}) = 1 - \int_0^1 C_{t|t-1}(u_t|u_{t-1};\alpha)du_t \ . \tag{9}$$

[2]Instead of using the rescaled empirical distribution function, one could use an adequate kernel estimator of the distribution function. Furthermore, they offer an appropriate bootstrap method to construct statistical inference procedures for the estimated quantiles.

Therefore, without loss of generality the identity mapping case yields to the autoregressive process[3]

$$u_t = \Lambda_2(u_{t-1}) + \nu_t \ . \tag{10}$$

Contrary to the traditional linear case, $|\alpha| < 1$,

$$u_t = \alpha u_{t-1} + \epsilon_t \tag{11}$$

with an iid error ϵ_t, $E(\epsilon_t|u_{t-1}) = 0$, the copula-based approach allows for nonlinear temporal dependence structures. In order to calculate a proxy for the systematic temporal dependence between u_{t-1} and u_t substitute the theoretical quantile u_t by its nonparametric estimate of the empirical distribution $\widehat{u}_t = \frac{n+1}{n}\widehat{G}(y)$ and name the ascendingly sorted empirical quantiles $\widehat{u}_t$ by $\widehat{u}_t^{\star\star}$. The systematic projection of the expected quantile in the linear case is $\widehat{u}_t^{\star} = \widehat{\alpha} \cdot \widehat{u}_{t-1}^{\star\star}$ and leads to a constant strength of temporal dependence calculated by $\Delta\widehat{u}_t^{\star} = \widehat{\alpha}/n$. In the generalized case the systematic projection of the expected quantile is

$$\widehat{u}_t^{\star} = C_{t|t-1}^{-1}(0.5|\widehat{u}_{t-1}^{\star\star}; \widehat{\alpha}) \tag{12}$$

and can be used to calculate the proxy for the strength of temporal dependence $\Delta\widehat{u}_t^{\star} = \widehat{u}_t^{\star} - \widehat{u}_{t-1}^{\star}$. This generalized version of temporal dependence allows also for nonlinear dependencies conditional on the level of $\widehat{u}_{t-1}^{\star}$ and the copula C. Therefore, the following definition of conditional temporal dependence will be considered:

Definition 1 *The proxy for conditional temporal dependence between the random variables U_{t-1} and U_t given $\widehat{u}_{t-1}^{\star}$ and a copula C is defined by:*

$$dep(U_{t-1}, U_t|\widehat{u}_{t-1}^{\star}, C) := \Delta\widehat{u}_t^{\star}$$

Once the values for $\Delta\widehat{u}_t^{\star}$ are calculated, every $\widehat{u}_t$ can be uniquely related to a value y_t and $\Delta\widehat{u}_t^{\star}$ and leads to a time series of conditional temporal dependencies $dep(Y_{t-1}, Y_t|y_{t-1}, C)$ which correspond to the values of y_t. Although the copula parameters - which can be transformed to the correlation coefficient according to Kendall or Spearman - are treated as time invariant (α and not α_t) the copula itself allows for a variation of temporal dependence conditional on the quantile level.

[3]Strictly speaking the process is an autoregressive quantile process, whereas the quantile treatment can be simply interpreted as a stabilizing transformation.

3 Model

Consider the random variable $Y^{\star}_{i,t}$ which stands for stock market uncertainty of investor $i = 1, \ldots, m$ at the end of the last trading day of month $t = 1, \ldots, n$ and its realization $y^{\star}_{i,t}$.[4] According to Aoki and Yoshikawa (2007) about 95 percent of all market participants consider two investment strategies. With two largest clusters, there are two regimes; one with a cluster of investors with strategy 1 as the largest share, and the other with a cluster of investors using strategy 2 as the largest share. Namely, fundamentalists dominate the market in regime 1 and chartists dominate the market in regime 2. We postulate that the decision at the end of period t of a market participant i being a fundamentalist ($y_{i,t} = 1$) or a chartist ($y_{i,t} = 0$) is determined by individual stock market uncertainty $y^{\star}_{i,t}$ and an individual threshold φ_i for being a chartist or a fundamentalist.

$$y_{i,t} = \begin{cases} 0 & , \text{ if } \; y^{\star}_{i,t} \geq \varphi_i \\ 1 & , \text{ if } \; y^{\star}_{i,t} < \varphi_i \end{cases}$$

This decision rule implies that individuals make their strategy decision once a month and know their own threshold φ_i.

The main argument for this decision rule is the attempt of the investors to maximize their expected profits. Consider stock market uncertainty in the conventional sense as expected stock market variability. Hence, $y^{\star}_{i,t}$ can be substituted by $E_{i,t}(\sigma_{t+1})$, where σ_{t+1} stands for stock market variability during the month $t + 1$. As Aoki and Yoshikawa (2007) show, a market structure dominated by chartists leads to higher stock market variability σ than a market structure dominated by fundamentalists. Corresponding to Fama (1970) the market structure dominated by chartists reflects inefficient markets and the market structure dominated by fundamentalists reflects weak efficient markets. It is therefore conceivable that investors conclude from variability to market efficiency and reflects a new argument in economics. This behavioral assumption allows for the link between $E_{i,t}(\sigma_{t+1})$ and $E_{i,t}(\text{market efficiency}_{t+1})$. In case of inefficient markets asset prices do not reflect historical price information and it is possible to earn excess returns r by being a chartist. On the other hand, if the market is rather weakly efficient, asset prices reflect historical price information and it is possible to achieve excess returns by being a fundamentalist. Consequently, $y^{\star}_{i,t} \geq \varphi_i$ implies $E_{i,t}(r_{t+1}|y_{i,t} = 0) > E_{i,t}(r_{t+1}|y_{i,t} = 1)$ and $y^{\star}_{i,t} < \varphi_i$ implies

[4]In fact $Y^{\star}_{i,t}$ symbolizes the quantile of stock market uncertainty. In order to avoid a burdensome notation the economic argumentation neglects this transformation without loss of generality in this section.

$E_{i,t}(r_{t+1}|y_{i,t}=1) > E_{i,t}(r_{t+1}|y_{i,t}=0)$. Hence, the investment decision is motivated by expected profits and follows the expected market structure. High uncertainty leads to an investment strategy which causes higher stock market variability (Aoki and Yoshikawa (2007)). Hence, the decision rule acts like an accelerator for financial market instability and resembles a game with strategic complementarities, which induces nonlinearities on a macroeconomic level (see Cooper (1999)).

To construct a proxy for stock market confidence it is necessary to formulate an expectation formation mechanism of the expectation $E_{i,t}(\sigma_{t+1}) = y^{\star}_{i,t}$ in the decision rule. The following rule is motivated by Keynes (1936) and explains expectations by a projection of the existing situation and expected changes. Adopting this general approach in a time series context the projection of the existing situation is $\Lambda_3(E_{i,t-1}(\sigma_t))$ with a function Λ_3 determined by a copula. The expected changes are $E_{i,t}(\sigma_{t+1}|\sigma^{-}_t) - E^{-}_t(\sigma_{t+1}) = \epsilon^{\star}_{i,t}$ with a projection of realized variability $E_{i,t}(\sigma_{t+1}|\sigma^{-}_t)$ conditional on information concerning realized variability σ^{-}_t up to the day prior the last trading day and information concerning market variability expectations $E^{-}_t(\sigma_{t+1})$ until the day before the last trading day. According to this thoughts we receive the individual expectation formation

$$y^{\star}_{i,t} = \Lambda_3(y^{\star}_{i,t-1}) + \epsilon^{\star}_{i,t} \tag{13}$$

with $E(\epsilon^{\star}_{i,t}|y^{\star}_{i,t-1}) = 0$. The variability $\sqrt{V(\epsilon^{\star}_{i,t}|y^{\star}_{i,t-1})} = |\epsilon^{\star}_{i,t}|$ of $\epsilon^{\star}_{i,t}$ corresponds to the absolute deviation of individual realized variability expectations and market variability expectations. Following Keynes (1936)[5] "confidence" is defined by the relevance - or equivalently weight - of the systematic expectation argument. Dependent on the state of confidence a specific expectation follows and is caused by confidence. Regarding equation (13) the systematic component $y^{\star}_{i,t-1}$ is weighted by the function Λ_3. If the relevance of $y^{\star}_{i,t-1}$ for $y^{\star}_{i,t}$ is high, the confidence of the expectation argument is high and vice versa. This mechanism implies in connection with the decision rule that in case of high confidence the development of expectations show more persistence and with it more persistence of the development of investment strategies. The market participants have less incentive to change their strategy in face of high confidence. As long as the expectations are linked to stock market variability it is reasonable to equate expectation confidence with stock market confidence. Hence, the correct specification of Λ_3 in the copula-based Markov approach of (13) allows for a description of stock market confidence dependent on the level of stock market uncertainty. In line with

[5]It is astonishing that Keynes already recognized the importance of confidence and that his work is relevant for currently unsolved problems.

Definition 1 individual stock market confidence is then measurable by the temporal dependence between $Y^\star_{i,t-1}$ and $Y^\star_{i,t}$. Leaving the individual level by aggregating individual investment decisions leads to the market structure $S_{t+1} = m^{-1}\sum_{i=1}^{m} y_{i,t}$ with $0 \leq S_{t+1} \leq 1$ and the market uncertainty $E_t(\sigma_{t+1}) = y^\star_t = m^{-1}\sum_{i=1}^{m} y^\star_{i,t}$, which can be described analog to (13) by

$$y^\star_t = \Lambda(y^\star_{t-1}) + \epsilon^\star_t \tag{14}$$

with $E(\epsilon^\star_t|y^\star_{t-1}) = 0$. The market wide stock market confidence proxy is the temporal dependence of the market wide stock market uncertainty.

Concerning the question whether monetary policy is able to influence stock market uncertainty in a causal manner, it is crucial to answer the question whether monetary policy is able to affect the uncertainty determining argument, here stock market confidence. In case of nonlinear dependence between uncertainty and its fundamental reason, any linear empirical investigation of causality between monetary policy and stock market uncertainty must fail. The concrete dependence between uncertainty and confidence is specified by investment behavior and is rather difficult to derive analytically. Hence, the time series approach offers a suitable proposal for the derivation of behavioral pattern on an applicable empirical basis.

4 Copula selection

A canonical proxy for stock market uncertainty is the volatility index VIX of the S&P 500 created by the Chicago Board Options Exchange (see e.g. Bloom (2009)). We use data from Thompson Datastream[6] for the period January 1990 to October 2010. Hence, the number of observed months is $n = 250$. The development of the VIX is shown in Figure 1. In order to derive a stock market confidence proxy four parametric copulas are discussed (Gauss, Clayton, Frank, Fang). Although, copula-based Markov approaches are not implemented in available software packages, technical details are relegated to the Appendix to emphasize the economic argumentation. The hypothesis that the Fang copula captures the time dependence structure of the VIX can not be rejected on any plausible level of significance. Based on empirical tests the correctness of the remaining copulas can be rejected. Hence, the Fang copula seems to be the only correct copula in the set of copulas. To test the correctness of a copula in a first-order Markov framework, consider the following multiple hypothesis test of interest (notation in line with section 2):

[6]The time series code for the daily VIX closing prices in US dollars is "CBOEVIX".

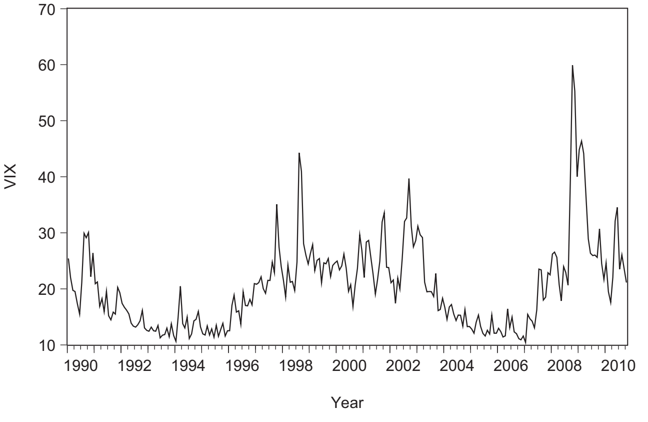

Figure 1: Monthly VIX closing prices.

H_0: $\{Y_t\}$ is a first-order Markov process with copula C

H_0 is equivalent to

H_0' : $V_t = C_{t|t-1}(U_t|U_{t-1};\alpha)$ is uniformly distributed on $[0,1]$ and not autocorrelated

We reject H_0 if H_0' is rejected. Table 1 shows the estimation and test results.

The nonparametric estimate of Spearman's correlation coefficient between U_{t-1} and U_t of 0.88 is similar to the Fang implied estimate of 0.81 according to equation (20) and the ML-estimates. This empirical fact supports the

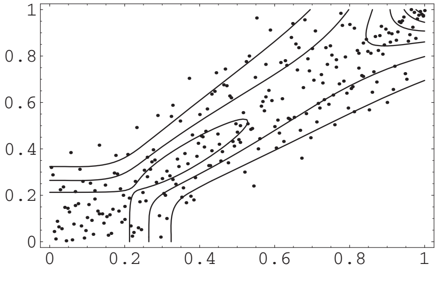

Figure 2: Contour with scatter plot of the empirical VIX quantiles t on the abscissa and $t+1$ on the ordinate of the Fang copula, with $\widehat{\alpha} = 0.175$ and $\widehat{\beta} = 0.9994$.

Table 1: VIX results

Copula	ML-estimation		Estimated autocorrelation				G-o-f
	α	β	1	2	3	4	
Gauss	0.849		-0.173*	0.036	-0.082	0.028	0.871
	(0.033)		(0.007)	(0.574)	(0.201)	(0.671)	(0.435)
Clayton	2.002		0.052	0.239*	0.067	0.115	1.274
	(0.118)		(0.418)	(0.000)	(0.298)	(0.072)	(0.078)
Frank	10.843		-0.130*	0.031	-0.063	0.046	0.632
	(0.790)		(0.040)	(0.625)	(0.323)	(0.475)	(0.819)
Fang	0.175	0.9994	-0.084	0.050	-0.030	0.051	0.622
	(0.015)	(0.0004)	(0.186)	(0.431)	(0.644)	(0.427)	(0.835)

Sample: 1990:1-2010:10 • Initial value of the one parameter copulas is 1 and of the Fang copula are $\widehat{\alpha}_1 = 0.4$ and $\widehat{\beta}_1 = 1$ • ML-estimates are different from zero at any level of significance (standard errors in brackets) • Spearman's correlation coefficients and p-values of the hypothesis $\rho_s(V_t, V_{t-l}) = 0$, $l = 1, 2, 3, 4$, in brackets • * indicates a significant autocorrelation on the 10% overall error rate using Bonferroni's adjustment (see e.g. (Sokal & Rohlf, 1995)) • 2 is the number of tests performed (correlation test up to a specific lag and goodness-of-fit (G-o-f) test) • Finite sample adjustment of the Kolmogorov statistic and corresponding p-values of the hypothesis $V_t \sim U[0, 1]$ in brackets (see e.g. (D'Agostino & Stephens, 1986))

correctness of the Fang copula.

To control the appropriateness of the Fang copula the Appendix contains a robustness check, which leads to the conclusion that the data obey tail dispersion. This tail dispersion can not be modelled by the Gauss and Clayton copula. Even the Frank copula as a representative of a copula with symmetric tail dispersion is inferior in comparison to the Fang copula. Only the Fang copula is able to deal with asymmetric tail dispersion. Summing up the hypothesis tests and the robustness checks the correctness of the Fang copula is indicated. Figure 2 allows for a graphical inspection of its density based on the parameter estimates. The Fang copula shows more density mass in the lower and upper tails and confirms the asymmetric tail dispersion issue.

5 Monetary policy, confidence and uncertainty

Once the correct copula is specified, it is possible to calculate the stock market confidence proxy according to equation (12) and Definition 1.[7] The left panel of Figure 3 points out the dependence structure between confidence and

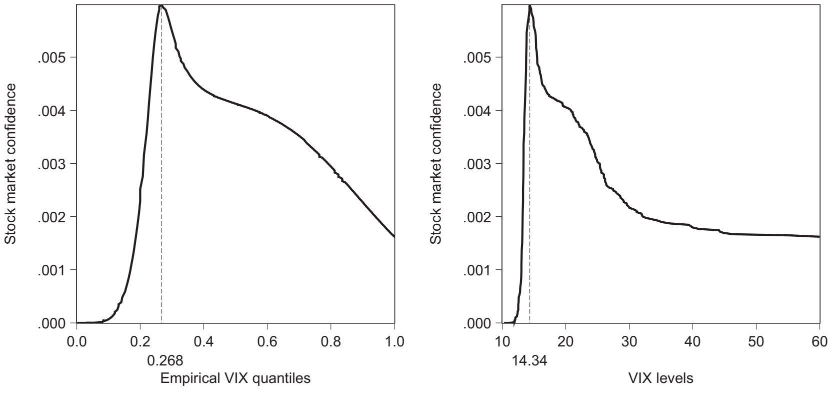

Figure 3: Dependence between US stock market uncertainty and US stock market confidence.

uncertainty quantiles and the right panel shows the dependence between confidence and uncertainty levels. The backward projection from the quantiles to the levels is done by the empirical distribution function. By using copula-based Markov models as the methodological framework the statistical significant and stable nonlinear relationship between the Keynesian motivated stock market confidence proxy and its dependent stock market uncertainty can be derived. If the monetary authority is able to influence stock market uncertainty, it must influence the uncertainty driving factor "confidence". To establish causal relationships in the Granger sense, an autoregressive framework with several variables is considered. In line with the literature (see e.g. Bekaert and Hoerova (2010)) the monetary policy stance is measured by the real interest rate in percent $r_t = i_t - \pi_t$, where i_t equals the monthly average

[7]Due to the fact that the inverse of the conditional distribution $C_{t|t-1}^{-1}$ does not exist in closed form, the empirical VIX quantile $\widehat{u}_t = C_{t|t-1}^{-1}(0.5|\widehat{u}_{t-1};\widehat{\alpha},\widehat{\beta})$ can be obtained from the equation $0.5 = C_{t|t-1}(\widehat{u}_t|\widehat{u}_{t-1};\widehat{\alpha},\widehat{\beta})$ using a numerical root-finding routine (here: Newton's procedure). Hence, numerical imprecisenesses of the root-finding routine can lead to obvious outliers and can be substituted by local means.

of the Federal Funds Rate in percent and $\pi_t = 12 \cdot \ln(cpi_t/cpi_{t-1}) \cdot 100$ stands for inflation in percent based on the seasonally adjusted consumer price index (1982-84=100) cpi.[8] Like Bekaert and Hoerova (2010) we account for business cycle variation and incorporate in our analysis the first difference Δe_t of the unemployment rate.[9] Let k_t denote monthly stock market confidence calculated on the basis of the Fang copula and its ML-estimates.[10] Furthermore, an intercept and some dummy variables are considered. One dummy $d_{crisis,t}$ accounts for the recent financial crisis and comprises the value 1 for the period 2007:8 to 2010:10 and 0 elsewhere. According to Figure 3 the effect of confidence on uncertainty is inverse and depends on whether a low uncertainty regime is observed or a high uncertainty regime. Every empirical VIX quantile smaller or equal than 0.268 generates the value 1 in the dummy $d_{low,t}$ and 0 elsewhere. To account for the dynamics $d_{low,t-1}$ is also included into the following autoregression, which is motivated by the Granger causality test.

$$k_t = \beta_1 + \beta_2 k_{t-1} + \beta_3 r_{t-1} + \beta_4 \Delta e_{t-1} + \beta_5 d_{crisis,t} + \beta_6 d_{low,t} + \beta_7 d_{low,t-1} + \varepsilon_t \quad (15)$$

The estimation results are shown in Table 2. In order to test whether mon-

Table 2: OLS estimation and Granger causality test

β_1	β_2	β_3	β_4	β_5	β_6	β_7
2.630**	0.333**	-0.060**	-0.693*	-0.570**	-2.357**	0.743**
(0.291)	(0.072)	(0.020)	(0.401)	(0.163)	(0.384)	(0.358)

Sample: 1990:3-2010:10 • ** and * indicate the rejection of the hypothesis of zero coefficients on the 95% and 90% level • White heteroskedasticity consistent standard errors in brackets • $R^2 = 0.46$ • p-values: Breusch-Godfrey test (lag 12) = 0.21 - ARCH(1) test = 0.11 - White test (cross terms) = 0.00 - Wald test ($\beta_3 = \beta_4 = 0$) = 0.001

etary policy does not cause stock market confidence it is necessary to test the hypothesis $\beta_3 = 0$. On any plausible level of significance this hypothesis will be rejected and it allows for the confirmation of causality. Analogously, it is also evident that the labor market causes confidence. Moreover, monetary policy and the labour market jointly ($\beta_3 = \beta_4 = 0$) cause stock market

[8]All necessary monthly data is obtained from Thompson Datastream. The appropriate Datastream codes are: i = "USFDFUND", cpi = "USCONPRCE"

[9]The Datastream code for the unemployment rate in levels is "USUN%TOTQ".

[10]The originally calculated values are multiplied by the factor 1000. This transformation leads to more feasible coefficient estimates in the following.

confidence on any plausible level of significance. This is a very important conclusion of the Granger causality tests. Monetary policy is able to influence expectations concerning stock market variability by real interest rates. This conclusion underlines the importance of the monetary authority during financial excess. On the other hand monetary policy is not the only causal argument for confidence, due to the causality effect of the labour market.

Remarkable is the negative sign of the highly significant estimate $\widehat{\beta}_5$, which identifies the current financial crisis as a heavy confidence crisis. This sensible result empirically confirms the adequacy of the confidence proxy. Regimes characterized by very low stock market uncertainty leads to a confidence drop ($\widehat{\beta}_6 < 0$). If the uncertainty persists on the low level the confidence drop will be partially compensated ($\widehat{\beta}_7 > 0$). Due to the construction of the dummy variables equation (15) is reducible to

$$k_t = \beta_1 + \beta_2 k_{t-1} + \beta_3 r_{t-1} + \beta_4 \Delta e_{t-1} + \epsilon_t \tag{16}$$

during high uncertainty regimes (neglecting the extraordinary current financial crisis).

By combining the results of the copula-based Markov approach and the autoregression the dependence structure between monetary policy and stock market uncertainty can be separated into a nonlinear and a linear part. Conditional on stock market confidence a specific level of stock market uncertainty follows in a nonlinear manner described by the Fang copula. On the other side stock market confidence is caused by monetary policy and the labour market in a linear manner. This issue leads to the conclusion that the stock market confidence proxy could be in general a useful tool in macroeconomic investigations concerning stock market uncertainty. Linear econometric methods are still appropriate even when the entire dependence structure between stock market uncertainty and macroeconomic variables seem to be nonlinear. To account for this fact the copula-based confidence proxy absorbs the nonlinear dimension of the problem. With respect to the real interest rate the following ceteris paribus reactions are derived:[11]

$$\text{Low uncertainty regime:} \quad VIX \leq 14.34 \Rightarrow \tfrac{\Delta VIX}{\Delta r} > 0 \tag{17}$$

$$\text{High uncertainty regime:} \quad VIX > 14.34 \Rightarrow \tfrac{\Delta VIX}{\Delta r} < 0 \tag{18}$$

Even in the ceteris paribus case the numerical monetary effect depends on the VIX level. For example, consider the case of unemployment stagnation ($e = 0$), an initial uncertainty level of approximately 23 and a real interest rate of 4%. Under these circumstances the development of stock market confidence

[11]The VIX threshold of 14.34 is obtained from Figure 3.

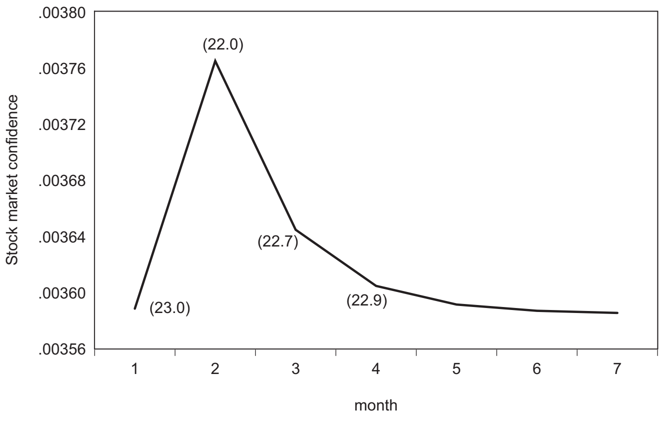

Figure 4: Stock market confidence response to a unique interest rate shock of minus 3 percentage points (corresponding VIX values in brackets). Initial situation in month 1: $r = 4\%$, $e = 0\%$, $VIX = 23$

is stable. Based on the parameter estimates of Table 2 a permanent interest rate reduction from 4% to 1% leads to a stable uncertainty level of 21.3 (ceteris paribus). In order to achieve the same uncertainty reaction without interest rate adjustments the unemployment rate has to be reduced ($\Delta e = -0.26\%$). In face of the mean value of 0.017% for unemployment changes, this unemployment adjustment seems to be large. The response of stock market confidence to a unique real interest rate shock of -3 percentage points is shown in Figure 4.[12] Interest rate shocks are temporary in nature and persist approximately one quarter.

6 Conclusions

This paper investigates the effect of monetary policy on stock market uncertainty. The uncertainty reaction is separable into a linear and nonlinear part. Motivated by a game with strategic complementarities nonlinearity is introduced on a macroeconomic level. Based on Keynesian expectation formation a proxy for stock market confidence is derived. According to this proxy stock market confidence is measurable as temporal dependence of stock

[12]The scaling of stock market confidence corresponds to the confidence scaling in Figure 3.

market uncertainty. The nonlinear dependence structure between confidence and uncertainty is modelled by a copula-based Markov approach. Nonlinear tail dispersion of the VIX data - which is interpreted as stock market uncertainty - is only captured by the copula of Fang et al. (2000). For low uncertainty regimes ($VIX \leq 14.34$) increasing confidence leads to increasing uncertainty. In case of high uncertainty regimes ($VIX > 14.34$) increasing confidence leads to decreasing uncertainty. Hence, the dependence structure between the uncertainty driving factor confidence and uncertainty is strongly nonlinear.

The linear effect of monetary policy on stock market confidence is confirmed by Granger causality tests. Real interest rates as the measure of monetary policy stance affect confidence in an inverse manner. Increasing interest rates lead to decreasing stock market confidence. Furthermore, real interest rate shocks persist approximately one quarter. A second causal argument for confidence is the labor market. Increasing unemployment changes lead to decreasing confidence. Moreover, the current financial market crisis can be identified as a heavy confidence crisis based on the confidence proxy.

Summing up the economic model and empirical results it is possible to conclude that causality runs from monetary policy and labor market conditions linear to stock market confidence and finally nonlinear to stock market uncertainty. Direct linear investigations between macroeconomic indicators and uncertainty neglect the uncertainty driving factor ,,confidence" and its nonlinear impact on uncertainty. In the light of the results of the paper insignificant effects of monetary policy on stock market uncertainty are the consequence of linear misspecifications. The monetary authority is therefore very well in the position to influence financial excess during financial crisis.

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Appendix

Copula review:

Bivariate tail dependence is one way to focus on variability of temporal dependence. This concept relates to the amount of dependence in the lower-quadrant tail or the upper-quadrant tail of a bivariate distribution (see e.g. (Joe, 1997)) and is relevant for dependence in extreme values. A copula has lower tail dependence if $\lambda_L \in (0, 1]$, where $\lambda_L = \lim_{u \to 0} P(U_{t-1} \leq u | U_t \leq u)$, and no lower tail dependence if $\lambda_L = 0$. Similarly, a copula has upper tail dependence if $\lambda_U \in (0, 1]$, where $\lambda_U = \lim_{u \to 1} P(U_{t-1} > u | U_t > u)$, and no upper tail dependence if $\lambda_U = 0$.

I. The Gauss copula (e.g. Joe (1997))

$$C(u_{t-1}, u_t; \alpha) = \Phi_\alpha[\Phi^{-1}(u_{t-1}), \Phi^{-1}(u_t)]$$

with the standard normal distribution function $\Phi(\cdot)$, the bivariate normal distribution function $\Phi_\alpha(\cdot, \cdot)$ with means zero and variances 1 and the correlation coefficient $|\alpha| < 1$ is an elliptical copula. Its lower tail dependence parameter is $\lambda_L = 0$ und its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits neither dependence in the negative tail nor in the positive tail. The copula density function $c(u_{t-1}, u_t; \cdot)$ is:

$$(1-\alpha^2)^{-1/2} \exp\left\{-\frac{1}{2}(1-\alpha^2)^{-1}[u_{t-1}^2 + u_t^2 - 2\alpha u_{t-1} u_t]\right\} \exp\left\{\frac{1}{2}[u_{t-1}^2 + u_t^2]\right\}$$

Due to the linearity of the Gauss copula according to Chen and Fan (2006) $\Phi^{-1}(u_t) = \alpha \Phi^{-1}(u_{t-1}) + \varepsilon_t$ with $\varepsilon_t \sim N(0; \sqrt{1-\alpha^2})$ follows. Consequently, $u_t = \Phi(\alpha \Phi^{-1}(u_{t-1}) + \varepsilon_t)$ and $v_t = \Phi(\varepsilon_t / \sqrt{1-\alpha^2})$ follows.

II. The Clayton copula (Clayton (1978))

$$C(u_{t-1}, u_t; \alpha) = \left(u_{t-1}^{-\alpha} + u_t^{-\alpha} - 1\right)^{-\frac{1}{\alpha}} \quad ,$$

$\alpha > 0$, is an asymmetric Archimedean copula. Its lower tail dependence parameter is $\lambda_L = 2^{-\frac{1}{\alpha}}$ und its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits greater dependence in the negative tail than in the positive tail. The copula density function is:

$$c(u_{t-1}, u_t; \alpha) = (1+\alpha)\,(u_{t-1}u_t)^{-\alpha-1}\left(u_{t-1}{}^{-\alpha} + u_t{}^{-\alpha} - 1\right)^{-2-1/\alpha}$$

The inverse of the conditional distribution is:

$$C_{t|t-1}^{-1}(v_t|u_{t-1}; \alpha) = u_t = [(v_t^{-\alpha/(1+\alpha)} - 1)u_{t-1}^{-\alpha} + 1]^{-1/\alpha}$$

III. The Frank copula (Frank (1979))

$$C(u_{t-1}, u_t; \alpha) = -\frac{1}{\alpha}\log\left(1 + \frac{(e^{-\alpha u_{t-1}} - 1)\,(e^{-\alpha u_t} - 1)}{(e^{-\alpha} - 1)}\right) \quad ,$$

$\alpha = (-\infty, +\infty)\backslash\{0\}$, is a symmetric Archimedean copula. Its lower tail dependence parameter is $\lambda_L = 0$ and its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits neither dependence in the negative tail nor in the positive tail and shows more tail dispersion than the Gauss copula. The copula density function is:

$$c(u_{t-1}, u_t; \alpha) = \alpha\eta e^{-\alpha(u_{t-1}+u_t)}/[\eta - (1 - e^{-\alpha u_{t-1}})(1 - e^{-\alpha u_t})]^2,\ \eta = 1 - e^{-\alpha}$$

The inverse of the conditional distribution is:

$$C_{t|t-1}^{-1}(v_t|u_{t-1}; \alpha) = u_t = -\alpha^{-1}\log\{1 - (1 - e^{-\alpha})/[(v_t^{-1} - 1)e^{-\alpha u_{t-1}} + 1]\}$$

In order to allow for a more flexible copula specification the following two parameter copula will be applied.

IV. The Fang copula (Fang et al. (2000))

$$C(u_{t-1}, u_t; \alpha, \beta) = \frac{u_{t-1}u_t}{\left[1 - \beta\left(1 - u_{t-1}{}^{\frac{1}{\alpha}}\right)\left(1 - u_t{}^{\frac{1}{\alpha}}\right)\right]^{\alpha}} \tag{19}$$

considers the parameters $\alpha > 0$ and $0 \leq \beta \leq 1$. When $\beta = 0$, U_{t-1} and U_t are independent. When $\beta = 1$, $C(u_{t-1}, u_t; \alpha, 1)$ in (19) becomes the bivariate Clayton copula. As $\alpha = 1$, $C(u_{t-1}, u_t; 1, \beta)$ is the Ali-Mikhail-Haq copula (Ali et al. (1978)) and the generalized Eyraud-Farlie-Gumbel-Morgenstern copula (Cambanis (1977)). By means of some stochastic transforms, some bivariate distributions can be induced by the Fang copula, such as the generalization of Gumbel's bivariate logistic distribution given by Satterthwaite and Hutchinson (1978). Moreover, it can be shown that if $\beta < 1$, $\lim_{\alpha \to 0} C(u_{t-1}, u_t; \alpha, \beta) = \lim_{\alpha \to \infty} C(u_{t-1}, u_t; \alpha, \beta) = u_{t-1}u_t$. Therefore, U_{t-1} and U_t are independent as $\alpha \to 0$ and $\alpha \to \infty$. To asses the correlation between two random variables, copulas can be used to define Spearman's ρ_s (see Joe (1997)) in general. Analog to the general case the Spearman's correlation coefficient of the Fang copula between U_{t-1} and U_t is representable by a hypergeometric function. A hypergeometric function of x is defined as

$$ {}_pF_q(a_1, \cdots, a_p; b_1, \cdots, b_q; x) = \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k} \frac{x^k}{k!} \ , $$

where $(a)_k = \Gamma(a+k)/\Gamma(a)$ and $a_1, \ldots a_p, b_1, \ldots, b_q$ are parameters. $\Gamma(z)$ stands for the gamma function $\int_0^\infty e^{-t} t^{z-1} dt$. Then, the Spearman's correlation coefficient $\rho_s(\alpha, \beta)$ of the Fang copula in (19) between U_{t-1} and U_t is given by

$$ \rho_s(\alpha, \beta) = 3({}_3F_2(1, 1, \alpha; 1 + 2\alpha, 1 + 2\alpha; \beta) - 1) \ . \tag{20} $$

The copula density function is:

$$ c(u_{t-1}, u_t; \alpha, \beta) = \frac{(\beta^2 + \beta/\alpha)(u_{t-1}u_t)^{1/\alpha} + (\beta - \beta^2)(u_{t-1}^{1/\alpha} + u_t^{1/\alpha}) + (1-\beta)^2}{[1 - \beta(1 - u_{t-1}{}^{1/\alpha})(1 - u_t{}^{1/\alpha})]^{\alpha+2}} $$

$C_{t|t-1}^{-1}$ does not exist in closed form. $u_t = C_{t|t-1}^{-1}(v_t | u_{t-1}; \alpha, \beta)$ can be obtained from the equation $v_t = C_{t|t-1}(u_t | u_{t-1}; \alpha, \beta)$ using a numerical root-finding routine (here: Newton's procedure).

Robustness check:

Consider the nonparametric estimated conditional quantiles $\widehat{u}_t$, which contain no information about a parametric copula. On the other hand if a parametric copula is selected, it is possible to calculate copula implied conditional quantiles which are used to construct a copula-based confidence interval of the conditional quantiles. Regarding the level of significance ϵ it follows for the upper interval bound

$$ \widehat{u}_{t,\overline{\epsilon}} = C_{t|t-1}^{-1}(1 - \epsilon/2 | \widehat{u}_{t-1}; \widehat{\alpha}) \tag{21} $$

and for the lower interval bound

$$ \widehat{u}_{t,\underline{\epsilon}} = C_{t|t-1}^{-1}(\epsilon/2 | \widehat{u}_{t-1}; \widehat{\alpha}) \ . \tag{22} $$

The „overall region" of Table 3 reports the estimated error rates for all conditional quantiles $\widehat{u}_t$, $t = 2, \ldots, n$. Therefore, given $\widehat{u}_t$, $\widehat{u}_{t,\overline{\epsilon}}$ and $\widehat{u}_{t,\underline{\epsilon}}$ copula-based error rates are:

$$ \widehat{\epsilon}_{overall} = 1 - \left(\frac{1}{n-1} \sum_{t=2}^{n} 1\{\widehat{u}_{t,\underline{\epsilon}} \leq \widehat{u}_t \leq \widehat{u}_{t,\overline{\epsilon}}\} \right) \tag{23} $$

Focusing the tails of the bivariate copula leads to further information about the copula adequacy. The calculation of the estimated error rates of the ,,lower region" of Table 3 is analog to (23), but only valid for lower $\widehat{u}_t$. We define the region for lower quantiles by $\widehat{u}_t < \pi$ with $\pi = 1/3$.[13] According to

$$\widehat{\epsilon}_{lower} = 1 - \left(\frac{1}{\underline{n}} \sum_{t=2}^{\underline{n}} 1\{\widehat{u}_{t,\underline{\epsilon}} \leq \widehat{u}_t \leq \widehat{u}_{t,\overline{\epsilon}} \text{ and } \widehat{u}_t < \pi\} \right) \tag{24}$$

the estimated error rate for the lower region are computed. Consequently, for the ,,upper region"

$$\widehat{\epsilon}_{upper} = 1 - \left(\frac{1}{\overline{n}} \sum_{t=2}^{\overline{n}} 1\{\widehat{u}_{t,\underline{\epsilon}} \leq \widehat{u}_t \leq \widehat{u}_{t,\overline{\epsilon}} \text{ and } \widehat{u}_t > 1 - \pi\} \right) \tag{25}$$

holds. $\underline{n}$ stands for the cases with $\widehat{u}_t < \pi$ and $\overline{n}$ for the cases with $\widehat{u}_t > 1-\pi$. Table 3 shows additionally the root mean squared error of the true and estimated error rates separated according to different regions. The Fang copula is also superior with respect to this criterion.

[13] Also for varying π similar error rates are observed.

Table 3: Estimated conditional quantile error rates of the VIX

Copula	Lower region		Upper region		Overall region	
	ϵ		ϵ		ϵ	
	0.10	0.05	0.10	0.05	0.10	0.05
Gauss	0.14	0.11	0.09	0.07	0.08	0.06
	(0.05)		(0.02)		(0.02)	
Clayton	0.22	0.17	0.07	0.04	0.10	0.07
	(0.12)		(0.02)		(0.01)	
Frank	0.10	0.01	0.13	0.07	0.10	0.03
	(0.03)		(0.03)		(0.02)	
Fang	0.07	0.04	0.11	0.02	0.09	0.04
	(0.02)		(0.02)		(0.01)	

Sample: 1990:1-2010:10 • The estimated conditional quantiles $\widehat{u}_t$ are computed by the empirical distribution. By assuming a certain parametric copula a level of significance ϵ determines a $(1-\epsilon)$ confidence interval of the nonparametric estimated conditional quantiles $\widehat{u}_t$. With respect to the inverse conditional distributions for the upper interval bound $v_t = 1 - \epsilon/2$ and for the lower bound $v_t = \epsilon/2$ holds. The unknown copula parameters are substituted by appropriate ML-estimates according to Table 1. • The copula specific numbers are the relative frequencies for the nonparametric estimated conditional quantiles outside the parametric confidence interval. The lower quantile region is defined by quantiles in a range of $(0; 1/3)$. For the upper quantile region $(2/3; 1)$ holds. • Root mean squared errors of the regions in brackets