Inattentive Voters and Welfare-State Persistence

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Abstract

Welfare-state measures often tend to persist even when they seem to have become suboptimal due to changes in the economic environment. This paper proposes an information-based explanation for the persistence of the welfare state. I present a structural model where rationally inattentive voters decide upon implementations and removals of social insurance. In this model, welfare-state persistence arises from disincentive effects of social insurance on attentiveness. The welfare state crowds out private financial precautions and with it agents’ attentiveness to changes in economic fundamentals. When welfare-state arrangements are pronounced, agents realize changes in economic fundamentals later and reforms have considerable delays.

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1 Introduction

It is a frequently expressed view that the political process features an asymmetry between the speed of implementations and the speed of removals of welfare-state arrangements. Reforms enhancing the size of the welfare state seem easily and quickly implemented while opposite reforms face stronger opposition. Welfare-state measures thus tend to persist. This paper offers an information-based explanation for such welfare-state persistence.

Many authors agree that the welfare state is persistent.¹ Lindbeck (2003, page 20) studies welfare-state dynamics and observes "certain asymmetries between the politics of expansion and retreat". Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) describe welfare-state persistence after the great depression. They observe that the great depression led to increased public intervention in the US, the UK, France, and Italy. After the economies had recovered, however, public intervention did not diminish. Brooks and Manza (2004) find similar patterns in welfare-state dynamics of several OECD countries at the end of the twentieth century. They summarize that "welfare states within most developed democracies appear quite resilient in the face of profound shifts in their national settings" (page 1).

The contribution of this paper is to offer a new explanation for welfare-state persistence which is based on the effects of the welfare state on attentiveness. Since the welfare state crowds out private financial precautions, it also reduces incentives to inform oneself about economic fundamentals such as life expectancy or invalidity risk. These fundamentals do not only influence private decisions on savings or insurance but also determine the optimal social choice regarding welfare-state arrangements.

The frequency with which people inform themselves about fundamentals depends on their level of private financial precaution and the incentives for private precaution depend on welfare-state arrangements. If the degree of social insurance is low (high), people engage much (little) in private financial activity such as savings. Therefore, they also in-

form themselves frequently (rarely) about fundamentals. Consequently, if initial welfare-state arrangements are low, a change in fundamentals is quickly noticed by a majority of society and translated into appropriate policies. By contrast, the political delay is long when welfare-state arrangements are pronounced.

This reasoning relies on the presence of information (or rationality) costs. Information costs can take the form of real resource costs, utility costs, or cognitive difficulties at any stage in the process between observing an information and the implementation of the appropriate response (Sims 2003; Mankiw and Reis 2010). Even with perfect information available, decisions may appear as if agents had imperfect information in the first place, for instance if agents choose not to use all information, have difficulties figuring out the appropriate response, or make mistakes while translating decisions into behavior.

The importance of informational imperfections in democratic decision making has been stressed by Downs (1957). Downs pointed out that even small information costs can lead voters to be rationally ignorant and cause pronounced uncertainty about issues important for the optimal vote. In political sciences, it is a common view that voters are usually poorly informed about relevant political measures, see e.g. Lupia (1994) and McDermott (1997). In economics, many papers have studied voting behavior under uncertainty, mostly theoretically.\(^2\)

In the model presented in this paper, optimal social choices depend on stochastic fundamentals. Voters have no incentive to inform themselves about these fundamentals for political purposes because the importance of any individual vote is negligible. But agents seek information about fundamentals in order to improve their private savings decision. The incentives to save and thus the incentives to inform oneself are, in turn, affected by social choices.

Agents have an exogenous and uncertain income stream and decide upon savings. Due to the absence of a private insurance market, there is a precautionary motive for savings. Agents face a risk of receiving no market income in future periods but the probability of this event is a random variable itself. Thus, the risk of receiving no market income is an unknown fundamental in the model which determines optimal savings.

\(^2\)See e.g. Feddersen and Pesendorfer (1997), Myerson (1998), Shottts (2006), Gershkov and Szentes (2009), and Taylor and Yildirim (2010).
In the political process, agents decide whether to vote in favor of a social insurance. Agents are ex ante identical such that there is no distributional motive of social insurance. However, there is potential demand for social insurance since agents have no access to a private insurance market. The stochastic income risk is also a determinant of the optimal social choice because it determines the future dependency ratio.

Next to the savings and voting choices, a third decision of agents is a costly and active choice whether to inform themselves about income risk. Doing so improves both the savings and the voting decision but agents only value the private benefit of improved savings and do not internalize the social benefit of their attentiveness. Thus, the information choice is only affected by the incentives for private savings which are weakened by social insurance.

There are two theoretical concepts for modelling costly and active information choice in a dynamic framework. In the theory of inattention (Sims 2003), agents decide on the precision of the information they acquire in any instant of time, while the theory of inattentiveness (Reis 2006a; Reis 2006b) models agents’ decisions on the timing of their infrequent acquirement of perfect information. In the model presented in this paper, the concept of inattentiveness is used.

Empirical support for the inattentiveness hypothesis is provided by Lusardi (1999) and Ameriks, Caplin, and Leahy (2003) who report survey evidence that respondents only infrequently react to news and update plans. Carroll (2003) and Mankiw, Reis, and Wolfers (2004) analyze survey data on expectations and find that news disseminate slowly throughout the economy. Alternatively, similar observations would be made if agents had full information but faced a cost of changing behavior. Mullainathan and Washington (2009) provide evidence that voters tend to process information in a biased way such as to confirm previous voting decisions. Experimental evidence suggests that such behavior is only given up when incentives are high enough (Festinger and Carlsmith 1959).

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3Indirect empirical support for the inattentiveness hypothesis is provided by many papers showing that inattentiveness helps to explain seemingly anomalous aggregate phenomena on financial markets (DellaVigna 2009; DellaVigna and Pollet 2009) and regarding macroeconomic dynamics (Ball, Mankiw, and Reis 2005; Reis 2006a; Reis 2006b).
Agents in the model are rationally inattentive, i.e. they inform themselves infrequently about the state of the world but, if so, perfectly. When not informing themselves, agents remain completely inattentive and receive no new information at all. The model economy shifts between two aggregate states of the world with different levels of income risk. I consider a situation where social insurance is socially beneficial in only one of the two states. When agents believe this one to be the current state of the world, they vote in favor of social insurance. When social insurance is implemented, private savings are lower and, consequently, agents remain inattentive for longer periods of time. As a result, the removal of social insurance when a change in the state has made it suboptimal takes, in expectation, longer than the implementation of social insurance after a change in the state that makes the welfare state optimal.

Other papers have proposed explanations for welfare-state persistence under perfect information. One branch of the literature relates the phenomenon to changes in people’s preferences. Lindbeck (1995) and Lindbeck and Weibull (1999) argue that welfare-state persistence is due to gradual changes in social norms regarding the perception of transfer recipients in society. In political sciences, welfare-state persistence is often attributed to changes in "policy preferences" (see e.g. Brooks and Manza 2004). Another line of argument builds on changes in distributional conflicts. Agell (2002) argues that welfare states resist the pressures of globalization because globalization not only increases the efficiency costs of the welfare state but also increases the distributional conflict so that some voters’ demand for welfare-state measures increases. Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) offer an explanation based on the effects of redistribution on the future income distribution. In their paper, persistence arises from the fact that even temporary welfare-state measures affect incentives in a way generating a distributional conflict in the future. This in turn generates a sustained demand for the continuation of the welfare state. Beetsma, Cukierman, and Giuliodori (2009) present a framework where a median voter bargains with a richer politician. Their explanation for welfare-state persistence is that temporary increases in taxes increase the bargaining power of the median voter who afterwards enforces increased redistribution.

In my model, preferences are stable and there is no distributional
conflict since agents are ex-ante identical. The persistence of the welfare state stems from the fact that it crowds out private financial precaution and with it attentiveness to changes in the environment. That a social-insurance system crowds out private financial precaution has been modeled by e.g. Rust and Phelan (1997). Empirical evidence supporting this hypothesis is provided by Bird (2001).

Some papers have studied voting on welfare-state measures under uncertainty about an underlying state of the world and a given information structure. For example, Dhami (2003) analyzes voting on redistribution in a model of representative democracy where voters have asymmetric but given information. Laslier, Trannoy, and van der Straeten (2003) and Hansen (2005) study majority-voting models of redistribution with imperfect information. In Dhami (2003) and Hansen (2005), the information structure is exogenously given, while in Laslier, Trannoy, and van der Straeten (2003), it is endogenous but taken as given by agents. By contrast, in the model presented in this paper, agents face an active information choice. Finally, the paper is related to the literature on the determination of social insurance in voting models, see Persson (1983) and Wright (1986).

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, the model is solved for individual decisions of agents. Section 4 describes the aggregate dynamics of the model. Section 5 concludes.

2 The Model

In the model, agents take intertemporal decisions under uncertainty. The economy is subject to two frictions. First, information is only available at a cost such that agents will rationalize on information. Second, there is a lack of a private insurance market such that there is a precautionary motive for savings and, in principle, demand for distortionary social insurance. In the political process, agents balance expected costs and benefits of social insurance based on their potentially imperfect information.

The model economy is an endowment economy which is populated by a mass-1 continuum of dynasties. A dynasty consists of an infinite stream of agents who live for two periods each. Each dynasty has one member in each generation. Generations overlap but do not interact with each other
due to the absence of capital and factor markets. However, generations are linked through the transmission of information. Specifically, each agent receives all information her dynasty has at the beginning of her life.

Thus, each generation $t$ consists of a mass-1 continuum of agents who live for two periods, $t$ and $t+1$. Agents maximize

$$E_{i,t}[U_{i,t}] = E_{i,t}[u(c_{i,t,t}) + \beta \cdot u(c_{i,t,t+1})] - \kappa \cdot d_{i,t},$$

(1)

where $U_{i,t}$ is the lifetime utility of agent $i$ in generation $t$, $c_{i,t,t}$ denotes this agent’s consumption in period $t$, and $c_{i,t,t+1}$ is consumption of the agent in period $t + 1$. $E_{i,t}$ denotes the statistical expectation operator conditional on information available to agent $i$ of generation $t$. $d_{i,t}$ is an indicator variable describing the choice of the agent whether to be attentive to new information. $\kappa$ is a fixed utility cost of acquiring new information.

This cost $\kappa$ can be understood as the cost of obtaining information, processing and interpreting it. It may arise because agents find the process annoying or frustrating. Reis (2006a) argues that while some information may be observed at little cost, the costs of understanding it and determining the optimal response can be quite substantial. Likewise this cost could be modelled as a resource cost capturing e.g. payments to a financial advisor or as opportunity costs of time.

A rate of time preference, $\beta$, is included in equation (1) for convenience. To facilitate the exposition, I impose the parameter restriction $\beta = 1$. This restriction does not affect the qualitative results of the paper because political choices do not apply to intertemporal transfers in this model.

For analytical tractability, I will use a specific functional form for period utility,

$$u(c_{i,t,t+h}) = 4 \cdot c_{i,t,t+h} - (c_{i,t,t+h})^2$$

(2)

where $h = 0, 1$.\footnote{It is common to assume linear-quadratic (Hassler, Storesletten, and Zilibotti 2003; Chen and Song 2009) or quasi-linear (Tabellini 2000; Borck 2007) preferences in dynamic political-choice models in order to ensure tractability.} This utility function exhibits linear marginal utility, $u'(c_{i,t,t+h}) = 4 - 2 \cdot c_{i,t,t+h}$, and constant curvature, $u''(c_{i,t,t+h}) = -2$. Under the model set-up described below, the maximal amount of consumption in a period is $c_{i,t,t+h} = 2$. Therefore, the utility function
exhibits positive and decreasing marginal utility for all relevant levels of consumption.

In the first period of their life, agents receive a deterministic gross income $y_{i,t,t}$ normalized to one,

$$y_{i,t,t} = 1. \quad (3)$$

Income in the second period of life is stochastic. With probability $1 - \pi_t$, a generation-$t$ agent will receive a gross income of 1 also in period $t + 1$. With probability $\pi_t$, agent $i$ of generation $t$ will receive an income of 0 in period $t + 1$,

$$y_{i,t,t+1} = \begin{cases} 1, & \text{prob. } 1 - \pi_t \\ 0, & \text{prob. } \pi_t \end{cases}. \quad (4)$$

In the following, I will refer to agents receiving a positive gross income in the second period as "lucky" while agents with zero income in the second period will be called "unlucky".

The risk of receiving no income in the second period of life, $\pi_t$, follows an exogenous stochastic process. In particular, $\pi_t$ can take two values, $\pi^h$ and $\pi^l$, $\pi^h > \pi^l$. Thus there are two states of the world, a "good" one with low income risk and a "bad" one where income risk is high.

State changes occur with an exogenous probability $\lambda < \frac{1}{2}$ at any period, i.e. they follow a Bernoulli process with Bernoulli parameter $\lambda$. Thus, the stochastic process for $\pi$ can be described as a two-state Markov process with transition matrix $\Lambda$, given by

$$\Lambda = \begin{bmatrix} 1 - \lambda & \lambda \\ \lambda & 1 - \lambda \end{bmatrix}. \quad (5)$$

Income risk in period $t$ is the same as $k$ periods ago when the number of state changes between these two periods, denoted by $N(t - k, t)$, is even. Note that $\pi_t$ is a generation-wide variable determining the risk for each member of generation $t$ to receive no income in period $t + 1$. This risk is the same for all members of the generation and is determined between periods $t - 1$ and $t$.

For agents, there are two ways to cope with income risk, private (precautionary) savings and social insurance. There is no private insurance market. Agents have the possibility to save at a gross interest rate of 1, i.e. generation $t$ agent $i$ can store an amount $s_{i,t}$ of his income from
period $t$ to period $t + 1$. Furthermore, each generation $t$ can decide to implement social insurance. If so, the government evens out income differences perfectly. Specifically, it collects incomes from all lucky agents and redistributes incomes equally among the members of the generation. Thus, the contribution of the lucky agents is $\tau_t = 1$ when there is social insurance. It is assumed that the amount of total resources is lower in the presence of social insurance. This may capture disincetive effects or government inefficiency, which is modeled in a short-cut way for simplicity. From every unit of contributions collected, the government can only redistribute $c < 1$ units. If a generation decides against social insurance, I will capture this formally as a contribution of zero, $\tau_t = 0$.

The implementation of social insurance by a generation applies to both periods of the generation’s life. In the first period, social insurance is a waste of resources since agents are still identical and thus pay the same contributions and receive the same transfer. However, in the second period, social insurance reduces income risk at the price of lower expected income. Formally, net income $x_{i,t,t}$ of an agent $i$ of generation $t$ in the first period of her life is given by

$$x_{i,t,t} = 1 - (1 - c) \tau_t$$

and net income $x_{i,t,t+1}$ in the second period of her life is given by

$$x_{i,t,t+1} = \begin{cases} 
1 - \tau_t + (1 - \pi_t) c \tau_t, & \text{prob. } 1 - \pi_t \\
(1 - \pi_t) c \tau_t, & \text{prob. } \pi_t 
\end{cases}$$

where $\tau_t$ is the contribution implemented by generation $t$ and can be either one or zero. Equations (6) and (7) capture both political environments. When there is social insurance, $\tau_t = 1$, then first-period net income is $x_{i,t,t} = c$ and net income in the second period is $x_{i,t,t+1} = (1 - \pi_t) c$ independent of the agent’s individual draw of gross income. In the absence of social insurance, $\tau_t = 0$, the agent receives net income $x_{i,t,t} = 1$ in period $t$ and his second-period net income is either 0 or 1.

Agent $i$ of generation $t$ faces the following budget constraint in her first period of life:

$$c_{i,t,t} + s_{i,t} \leq x_{i,t,t}.$$
Thus, consumption and savings may not succeed her net income. In the second period consumption may not exceed net income plus savings,

\[ c_{i,t,t+1} \leq x_{i,t,t+1} + s_{i,t}. \] (9)

Political choices are decided by direct democracy. Each generation \( t \) decides upon whether to implement social insurance, i.e. \( \tau_t = 1 \), or not, i.e. \( \tau_t = 0 \), by a direct vote over these two opportunities. The vote takes place in a general, free, and secret ballot. All agents in generation \( t \) participate in this vote. Furthermore, I assume that agents vote truthfully in the sense that they vote for their individual expected-utility maximizing \( \tau_t \).\(^5\) The vote of agent \( i \) of generation \( t \) is denoted by \( \tau_{i,t} \in \{0, 1\} \).

Individual and public choices depend on income risk \( \pi_t \). However, agents can not costlessly monitor the process determining \( \pi_t \). In any period \( t \), agents can decide to obtain perfect information about \( \pi_t \) and to accept a fixed utility cost \( \kappa \). If an agent decides not to obtain the information, she will be said to be inattentive. Every agent transmits the information she has to the next member of her dynasty.

The time structure within periods is as follows. Prior to period \( t \), income risk \( \pi_t \) for generation \( t \) is determined according to the transition matrix (5). In this period \( t \), an agent of generation \( t \) first receives information from the member of her dynasty in generation \( t - 1 \). Second, she takes part in the referendum on the implementation of social insurance of her generation. Third, the agent decides whether or not to obtain complete information on income risk \( \pi_t \). Fourth, the agent receives net income \( x_{i,t,t} \), decides how much to save, and consumes the remaining part of her income.

In the second period of her life, the agent first bequeaths information to a member of generation \( t + 1 \). After this, she observes and receives her net income \( x_{i,t,t+1} \), and consumes. The timing of events is illustrated in Figure 1.

Agents’ decisions are determined by (potentially perfect) beliefs about the state of the world. Since agents have the possibility to update their beliefs, one has to distinguish between prior and posterior

\(^5\)Since any single voter has zero mass in this model, I abstain from analyzing strategic voting behavior and assume "sincere" (Barse, Cardak, Glomm, and Ravikumar 2009) or "naive" (Feddersen and Pesendorfer 1997) voting instead.
Figure 1: The timing of events and beliefs.
beliefs. Posterior and prior beliefs are labeled by different time indices. The time index \( t^+ \) refers to beliefs after the updating decision in period \( t \), whereas the time index \( t \) refers to the time in period \( t \) before the updating decision. An agent’s prior belief can be represented by the probabilities the agent assigns to the two possible states of the world, \( p_{i,t}^h = \text{prob}_{i,t} \left[ \pi_t = \pi^h \right] \) and \( 1 - p_{i,t}^h = \text{prob}_{i,t} \left[ \pi_t = \pi^l \right] \), where \( \text{prob}_{i,t} \left[ \cdot \right] \) denotes the probability of the event in the brackets conditional on information available to agent \( i \) of generation \( t \) before the updating decision. Analogously, \( p_{i,t^+}^h = \text{prob}_{i,t^+} \left[ \pi_t = \pi^h \right] \) denotes the agent’s posterior belief. When the agent decides to be attentive, she will know the state of the world for sure after updating, i.e. \( p_{i,t^+}^h = 1 \) or \( p_{i,t^+}^h = 0 \) then. By contrast, when the agent decides to be inattentive, then \( p_{i,t^+}^h = p_{i,t}^h \) and the posterior belief can take any value between zero and one. The timing of the belief formation can be seen from Figure 1.

To summarize, a formal description of the decision problem is as follows: Agent \( i \) of generation \( t \) chooses \( \tau_{i,t} \in \{0,1\} \), \( d_{i,t} \in \{0,1\} \), \( s_{i,t} \in [0,x_{i,t}] \) sequentially such as to maximize (1) subject to (8), (9), the information constraints described above and as if \( \tau_t = \tau_{i,t} \) capturing the sincerity of the voting decision.

Thus the decision problem of an individual agent includes three decision stages and a first stage where prior beliefs are determined. From stage to stage, the information set of the agent can change. Since I will solve the problem by backward induction, I summarize the different stages starting with the last one in the temporal ordering but the first one to be solved:

1. **Savings decision:** At the final stage, the agent chooses \( s_{i,t} \) to maximize \( E_{i,t} | U_{i,t} \) based on (potentially perfect) posterior beliefs \( p_{i,t^+}^h \) about the state of the world and knowing whether there is social insurance or not. Potential updating costs are sunk at this stage.

2. **Updating decision:** At this stage, the agent chooses whether to update information in order to maximize expected indirect utility (taking into account optimal subsequent savings) having some prior beliefs \( p_{i,t}^h \) about the state of the world and knowing whether there is social insurance. When the agents decides not to update, prior and posterior beliefs are identical, \( p_{i,t^+}^h = p_{i,t}^h \).
3. **Voting decision:** At this decision stage, the agent decides whether to vote in favor of social insurance in order to maximize expected indirect utility (taking into account optimal subsequent updating and savings) having some *prior beliefs* $p^h_{i,t}$ about the state of the world.

4. **Belief formation:** Prior to all decisions, the agent calculates subjective probabilities of the two states of the world, $p^h_{i,t}$ and $1 - p^h_{i,t}$, based on the received information.

3 Individual Decisions

In this section, I derive the decisions of an agent $i$ of generation $t$, in short agent $(i, t)$. Decisions of an agent depend only on her beliefs $p^h_{i,t}$ and $p^h_{i,t+}$. Thus agents with identical beliefs make identical decisions. This is the case because income in the second period of life, which is a source of heterogeneity, realizes after all decisions are taken.

3.1 Savings decision

When deciding upon individual savings, $s_{i,t}$, an agent $i$ of generation $t$ knows whether her generation has implemented social insurance. The agent furthermore has some belief about the current level of income risk. Since the updating decision has already taken place at this stage, the relevant belief is the posterior belief $p^h_{i,t+}$. If the agent has decided to be attentive, she knows the value of $\pi_t$ for sure, i.e. $p^h_{i,t+} = 1$ or $p^h_{i,t+} = 0$. If the agent has decided to remain inattentive to news, the belief is uncertain and reflects the received information about past income risk and its precision as a signal about current income risk.

At this stage, updating costs are already sunk. The agent seeks to maximize

$$E_{i,t+} \bar{U}_{i,t} = E_{i,t+} [u(c_{i,t,t}) + u(c_{i,t,t+1})],$$

(10)

which defines $\bar{U}_{i,t}$, based on the posterior belief $p^h_{i,t+}$ by choosing individual savings, $s_{i,t}$, subject to the two period budget constraints (8) and (9). Substituting constraints, the decision problem at this stage can be written as

$$\max_{s_{i,t}} E_{i,t+} [u(x_{i,t,t} - s_{i,t}) + u(x_{i,t,t+1} + s_{i,t})].$$
In this expression, \( x_{i,t,t+1} \) is stochastic and can take four values: If the state of the world is good, i.e. \( \pi_t = \pi^l \), the agent can end up lucky and her net income in period \( t + 1 \) is

\[
x_{i,t,t+1} = x^{l,L} = 1 - \tau_t + (1 - \pi^l) e \tau_t.
\]

(11)

However, the agent can end up unlucky even if the state of the world is good, then

\[
x_{i,t,t+1} = x^{l,U} = (1 - \pi^l) e \tau_t.
\]

(12)

If the state of the world is bad, i.e. \( \pi_t = \pi^h \), lucky agents end up with a net income of

\[
x^{h,L} = 1 - \tau_t + (1 - \pi^h) e \tau_t,
\]

(13)

while unlucky agents receive

\[
x^{h,U} = (1 - \pi^h) e \tau_t
\]

(14)

in this case.\(^6\)

Expected utility depends on the probabilities the agent assigns to these four scenarios. For instance, the agent believes to receive \( x^{l,L} \) with the probability of having luck conditional on the state of the world being good multiplied with the probability the agent assigns to the good state of the world,

\[
\text{prob}_{i,t+} \left[ x_{i,t,t+1} = x^{l,L} \right] = \text{prob}_{i,t+} \left[ y_{i,t,t+1} = 1| \pi_t = \pi^l \right] \cdot \text{prob}_{i,t+} \left[ \pi_t = \pi^l \right] = (1 - \pi^l) \cdot (1 - p^h_{i,t,t+}).
\]

(15)

Analogously, we can calculate

\[
\begin{align*}
\text{prob}_{i,t+} \left[ x_{i,t,t+1} = x^{l,U} \right] &= \pi^l \cdot (1 - p^h_{i,t,t+}), \\
\text{prob}_{i,t+} \left[ x_{i,t,t+1} = x^{h,L} \right] &= (1 - \pi^h) \cdot p^h_{i,t,t+}, \\
\text{prob}_{i,t+} \left[ x_{i,t,t+1} = x^{h,U} \right] &= \pi^h \cdot p^h_{i,t,t+}.
\end{align*}
\]

(16)

(17)

(18)

If the agent has chosen to be attentive, two of these probabilities (either (15) and (16) or (17) and (18)) are zero.

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\(^6\)Equations (11) to (14) simplify in both political regimes. If there is social insurance, then \( x^{l,L} = x^{l,U} = (1 - \pi^l) e \) and \( x^{h,L} = x^{h,U} = (1 - \pi^h) e \). By contrast, in the absence of social insurance it holds that \( x^{l,U} = x^{h,U} = 0 \) and \( x^{l,L} = x^{h,L} = 1 \).
Using these subjective probabilities, the decision problem becomes

$$\max_{s_{i,t}} u (1 - (1 - e) \tau_t - s_{i,t}) + \left[ (1 - \pi^l) \cdot (1 - p^h_{i,t+}) \cdot u (x^{l, L} + s_{i,t}) + \pi^l \cdot (1 - p^h_{i,t+}) \cdot u (x^{l, U} + s_{i,t}) + (1 - \pi^h) \cdot p^h_{i,t+} \cdot u (x^{h, L} + s_{i,t}) + \pi^h \cdot p^h_{i,t+} \cdot u (x^{h, U} + s_{i,t}) \right] .$$

The first-order condition for this problem is

$$u' (1 - (1 - e) \tau_t - s_{i,t}) = \left[ (1 - \pi^l) \cdot (1 - p^h_{i,t+}) \cdot u' (x^{l, L} + s_{i,t}) + \pi^l \cdot (1 - p^h_{i,t+}) \cdot u' (x^{l, U} + s_{i,t}) + (1 - \pi^h) \cdot p^h_{i,t+} \cdot u' (x^{h, L} + s_{i,t}) + \pi^h \cdot p^h_{i,t+} \cdot u' (x^{h, U} + s_{i,t}) \right]$$

(19)

which is a consumption Euler equation for the case where the product of the rate of time preference and the gross interest rate is one. Marginal utility in the first period then equals expected marginal utility in the next period.

For the functional form of period utility (2), the consumption Euler equation can be solved analytically for savings. The optimal amount of savings for agent $i$ of generation $t$ equalizes expected consumption in the two periods.

In generations without social insurance, i.e. for $\tau_t = 0$, optimal savings,

$$s_{i,t | \tau_t = 0} = \frac{\bar{\pi}^e_{i,t+}}{2},$$

(20)

depend only on expected income risk $\bar{\pi}^e_{i,t+} = (1 - p^h_{i,t+}) \cdot \pi^l + p^h_{i,t+} \cdot \pi^h$. Savings increase with the expected income risk $\bar{\pi}^e_{i,t+}$ which reflects the precautionary motive of savings. When generation $t$ has decided in favor of social insurance, i.e. for $\tau_t = 1$, optimal savings are given by

$$s_{i,t | \tau_t = 1} = \frac{e \cdot \bar{\pi}^e_{i,t+}}{2}$$

(21)

and depend on the level of government efficiency $e$. It is important that, since $e < 1$, savings are lower when there is social insurance. This implies that having better information when choosing savings has a smaller impact on lifetime utility in the presence of social insurance.\footnote{Equations (20) and (21) are derived in Appendices A.1 and A.2, respectively.}
At the updating decision, the agent takes into account the optimal subsequent savings behavior. Therefore it is useful to determine expected indirect lifetime utility net of updating costs which is determined by the solution to the optimization problem for savings. This expected indirect utility is a function of individual beliefs and the political regime. Individual beliefs are perfectly described by the probability assigned to the bad state, $p^h_{i,t+}$, and the political regime is perfectly described by the contribution rate $\tau_t$. It is not necessary to distinguish between the states "attentive" and "inattentive" because the state "attentive" is a special case where $p^h_{i,t+}$ is either one or zero. I will denote expected indirect lifetime utility net of updating costs by

$$V(p^h_{i,t+}) := E_{i,t+} \left[ U_{i,t} \mid \tau_t = 0 \right]$$

for the case of no social insurance and

$$W(p^h_{i,t+}) := E_{i,t+} \left[ \tilde{U}_{i,t} \mid \tau_t = 1 \right]$$

for the case of social insurance.

Expected indirect lifetime utilities net of updating costs, $V$ and $W$, are derived in the following way. First, the optimal savings decision (20), or (21) respectively, together with the budget constraints (8) and (9) are used to determine the respective consumption levels in period $t$ and the possible consumption levels in period $t + 1$. Since consumption in period $t + 1$ is stochastic, the subjective probabilities (15) to (18) are used to determine expected utility in period $t + 1$. Finally, expected lifetime utility net of updating costs is given by the sum of the expected period utilities, according to equation (10).

When there is no social insurance, i.e. $\tau_t = 0$, expected indirect lifetime utility is given by

$$V(p^h_{i,t+}) = 6 - 3\pi^e_{i,t+} + \frac{(\pi^e_{i,t+})^2}{2}$$

(22)

and decreases in expected income risk. In the other political state, i.e. with social insurance, $\tau_t = 1$, expected indirect lifetime utility is

$$W(p^h_{i,t+}) = 8e - 2e^2 - 2\pi^e_{i,t+}e + \frac{(\pi^e_{i,t+})^2}{2}e^2 - e^2 \left[ E_{i,t+} (\pi_t)^2 \right]$$

(23)

where $E_{i,t+} (\pi_t)^2 = (\pi^l)^2 + p^h_{i,t+} \left( (\pi^b)^2 - (\pi^l)^2 \right)$. Here, expected indirect utility includes an expectation of the squared income risk because also net income in period $t + 1$ depends on $\pi_t$, see equation (7).\(^8\)

\(^8\)Equations (22) and (23) and their derivatives are derived in Appendices B.1 and
Three properties of the expected indirect utility functions are important for the subsequent analysis. First, both expected indirect utility functions (22) and (23) are convex in \( p_{i,t+}^{h} \), which can take any value between zero and one,

\[
\tilde{V}''(p_{i,t+}^{h}) = (\pi^{h} - \pi^{l})^{2} > 0, \tag{24}
\]

\[
\tilde{W}''(p_{i,t+}^{h}) = e^{2} (\pi^{h} - \pi^{l})^{2} > 0. \tag{25}
\]

The convexity implies that there are potential gains from updating because, when knowing \( \pi_{t} \) for sure, i.e. \( p_{i,t+}^{h} = 0 \) or \( p_{i,t+}^{h} = 1 \), agents can choose the appropriate savings level and thus improve relative to uncertain income risk.

Second, (23) is less convex than (22), in the sense that \( \tilde{W}''(p_{i,t+}^{h}) < \tilde{V}''(p_{i,t+}^{h}) \). In the presence of social insurance, agents save less and, consequently, the impact of an optimal savings decision on utility is lower. This implies that gains from updating are smaller when there is social insurance.

Third, there are constellations where agents would prefer social insurance only in one state of the world and not in the other, i.e.

\[
\tilde{V}(0) > \tilde{W}(0), \quad \tilde{V}(1) < \tilde{W}(1) \tag{26}
\]

or

\[
\tilde{V}(0) < \tilde{W}(0), \quad \tilde{V}(1) > \tilde{W}(1). \tag{27}
\]

Since the focus of this paper is on changes between political regimes, I will restrict the analysis to cases where either condition (26) or condition (27) is satisfied. It depends on the parameterization whether the agent is better off with social insurance when income risk is high or when it is low.\(^9\) Increases in income risk have two counteracting effects on the attractiveness of social insurance. First, rising income risk increases the probability that the agent will be a beneficiary of the social-insurance system and thus makes this welfare-state measure more attractive. Second, rising income risk also affects the dependency ratio decreasing the benefits the agent receives if unlucky in the second period, this second

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\(^9\)In Appendix B.3, I present examples for both, conditions (26) and (27), demonstrating that both these constellations exist.
Figure 2: Expected indirect utilities net of updating costs from optimal savings in the two political regimes (satisfying condition (27)).

effect makes social insurance less attractive. The results of the paper do not depend on which of the two effects dominates. The following illustrations given in figures and examples will refer to the case where condition (27) is fulfilled. In this case, agents are better off with social insurance when income risk is low.

When condition (26) or (27) is fulfilled, there is a unique posterior belief $p^*_+$ where the agent is in expectations equally off in both political regimes. Since $\bar{V}$ and $\bar{W}$ are strictly convex, there are at most two intersections between the two functions. When condition (26) or condition (27) is satisfied, the number of intersections between $\bar{V}$ and $\bar{W}$ on $(0,1)$ is odd. Together, this implies that the two functions intersect exactly once on $(0,1)$. Two expected utility functions (22) and (23) fulfilling condition (27) are illustrated graphically in Figure 2.
3.2 Updating decision

The agent will update her information whenever her expected indirect utility is higher when doing so. The agent enters this stage of the decision problem with knowledge about the political regime and a prior belief $p_{i,t}^h$ about income risk. In both political regimes, the decision whether to update will depend on the prior belief about income risk. When taking the updating decision, the agent takes into account optimal subsequent behavior.

Figure 3 illustrates the solution of the updating decision for the case of $\tau_t = 0$. The agent decides whether to update based on her prior beliefs about income risk, $p_{i,t}^h$. When the agent decides not to update, she will choose a savings level according to her prior belief. Consequently, the agent will then expect to receive a lifetime utility of $\tilde{V}(p_{i,t}^h)$ since $p_{i,t+1}^h = p_{i,t}^h$ and $d_{i,t} = 0$.

When the agent decides to be attentive, she will know $\pi_t$ for sure after updating, i.e. $p_{i,t+1}^h = 0$ or $p_{i,t+1}^h = 1$. The agent will then choose savings individually optimally according to the true income risk. However, in case the agent updates, her lifetime utility is reduced by the updating
cost $\kappa$. She will then receive either $\tilde{V}(0) - \kappa$ or $\tilde{V}(1) - \kappa$. Prior to updating, the agent expects to observe $\pi_t = \pi^h$ with probability $p_{i,t}^h$ and $\pi_t = \pi^l$ with probability $1 - p_{i,t}^h$. Before updating, the agents thus expect a lifetime utility level of $(1 - p_{i,t}^h) \cdot \tilde{V}(0) + p_{i,t}^h \cdot \tilde{V}(1) - \kappa$ in case she updates.

Since $\tilde{V}$ is convex in $p_{i,t-1}^h$, there are potential gains from updating. The agent will decide to update whenever

$$(1 - p_{i,t}^h) \cdot \tilde{V}(0) + p_{i,t}^h \cdot \tilde{V}(1) - \tilde{V}(p_{i,t}^h) > \kappa. \quad (28)$$

Updating costs could be that large that condition (28) would never be fulfilled. However, if there is some $p_{i,t}^h \in (0, 1)$ for which condition (28) is fulfilled, then there is a unique updating range between $\tilde{p}^0$ and $\tilde{p}^0$, due to the strict convexity of $\tilde{V}$. Whenever $\tau_t = 0$ and $p_{i,t}^h \in (\tilde{p}^0, \tilde{p}^0)$, the agent decides to be attentive and to obtain perfect information about income risk.

In the other political regime, $\tau_t = 1$, the updating decision works equivalently. Here, the agent updates whenever

$$(1 - p_{i,t}^h) \cdot \tilde{W}(0) + p_{i,t}^h \cdot \tilde{W}(1) - \tilde{W}(p_{i,t}^h) > \kappa. \quad (29)$$

If there is some $p_{i,t}^h \in (0, 1)$ for which condition (29) is fulfilled, then there is a unique range $(\tilde{p}^1, \tilde{p}^1)$ for which (29) is fulfilled since also $\tilde{W}$ is strictly convex.

Due to the constant second derivatives of both $\tilde{V}$ and $\tilde{W}$, both updating ranges, if they exist, are symmetric around 1/2. This implies that $\tilde{p}^0 = 1 - \tilde{p}^0$ and $\tilde{p}^1 = 1 - \tilde{p}^1$. This symmetry is the reason why it is not important whether agents prefer social insurance for high or low levels of income risk. The length of the range of beliefs for which the agent remains inattentive depends on the political regime but not on the specific end of the belief support. For instance, in the presence of social insurance, the agent chooses not to update for beliefs in $(0, \tilde{p}^1)$ and for beliefs in $(1 - \tilde{p}^1, 1)$. Both ranges have length $\tilde{p}^1$.

It is important that the updating range is smaller in the presence of social insurance which is crucial for the different information choices in the two political regimes. This results reflects that, when $\tau_t = 1$, savings are lower and thus choosing savings based on better information has a lower influence on lifetime utility. To show this result formally, note
that also the difference function \( \tilde{V} - \tilde{W} \) is convex in \( p_{i,t}^{h} \) since \( \tilde{V}''(p_{i,t}^{h}) > \tilde{W}''(p_{i,t}^{h}) \), see equations (24) and (25). Thus, the left hand side of (28) is always larger than the left hand side of (29). Therefore, whenever \( p_{i,t}^{h} \) fulfills condition (29), condition (28) is also fulfilled. However, there are values of \( p_{i,t}^{h} \) where condition (28) is fulfilled but not condition (29).

Furthermore, there a values of the information cost \( \kappa \) such that condition (28) has a solution on \((0,1)\) but condition (29) has not. If this is the case, the agent would never update on income risk when social insurance is implemented but sometimes do so when there is no social insurance. When condition (28) is not fulfilled for any \( p_{i,t}^{h} \in (0,1) \), then also condition (29) is not fulfilled for any \( p_{i,t}^{h} \).

At the voting stage of the decision problem, the agent takes into account optimal subsequent behavior including optimal updating. Therefore, it is useful to determine the expected indirect utility function which arises from optimal savings and optimal updating. I denote this function as \( V(p_{i,t}^{h}) := E_{i,t}[U_{i,t} \mid \tau_t = 0] \) for the case of \( \tau_t = 0 \) and \( W(p_{i,t}^{h}) := E_{i,t}[U_{i,t} \mid \tau_t = 1] \) for the case of \( \tau_t = 1 \). In the absence of social insurance, this function is

\[
V(p_{i,t}^{h}) = \begin{cases} 
\tilde{V}(p_{i,t}^{h}), & p_{i,t}^{h} \notin (p^0,\bar{p}^0), \\
(1 - p_{i,t}^{h}) \cdot \tilde{V}(0) + p_{i,t}^{h} \cdot \tilde{V}(1) - \kappa, & p_{i,t}^{h} \in (p^0,\bar{p}^0).
\end{cases}
\]

Analogously, in the presence of social insurance, expected lifetime utility as a function of the agent’s belief is

\[
W(p_{i,t}^{h}) = \begin{cases} 
\tilde{W}(p_{i,t}^{h}), & p_{i,t}^{h} \notin (\underline{p}^1,\bar{p}^1), \\
(1 - p_{i,t}^{h}) \cdot \tilde{W}(0) + p_{i,t}^{h} \cdot \tilde{W}(1) - \kappa, & p_{i,t}^{h} \in (\underline{p}^1,\bar{p}^1).
\end{cases}
\]

Two expected indirect utility functions \( V \) and \( W \) fulfilling condition (27) are illustrated graphically in Figure 4. \( V \) and \( W \) have a unique intersection \( p^* \) on \((0,1)\). In the constellation chosen in the figure, this intersection lies in the updating range.\(^{10}\)

### 3.3 Voting decision

At this stage, the agent decides whether to vote in favor of social insurance. She takes this choice such as to maximize expected indirect

\(^{10}\)This does not necessarily have to hold but is possible which is illustrated in Appendix B.3.
utility. She thereby takes into account optimal subsequent updating and savings. When entering this stage, the agent has some prior beliefs \( p^h_{i,t} \) about the state of the world.

Since voting for one or the other alternative is costless, the voting decision is rather simple to determine. The agent votes for the political system under which expected indirect utility is higher depending on the agent’s prior belief about the state of the world. Agent \((i,t)\) votes in favor of social insurance whenever

\[
W(p^h_{i,t}) > V(p^h_{i,t})
\]

and votes against it when \( W(p^h_{i,t}) < V(p^h_{i,t}) \).

Revisiting the expected indirect utility functions \( V \) and \( W \), it follows that there is a unique \( p^* \) for which the agent is indifferent between the two political regimes, see Figure 4. The voting decision is determined by whether the agent’s prior belief \( p^h_{i,t} \) is below or above \( p^* \). Whether
she votes in favor of social insurance when \( p_{i,t}^h > p^* \) or when \( p_{i,t}^h < p^* \) depends on the parametrization. However, the voting decision changes when the prior belief passes \( p^* \).

### 3.4 Belief formation

The prior belief \( p_{i,t}^h \) determines all decisions of the agent. Here I determine how this belief is formed given the information received from the previous generation. Agent \((i, t)\) receives all information her ancestor \((i, t - 1)\) had at the beginning of period \( t \). Agent \((i, t - 1)\) in turn received all information from agent \((i, t - 2)\) and so on. Consequently, agent \((i, t)\) knows the time of her dynasty’s last update on income risk and knows what the respective member observed at that time.

Consider an agent \((i, t)\) whose dynasty’s last update on \( \pi \) was in period \( t - j \). In period \( t \), the probability that income risk is still the same as at the time of the last update equals the probability that the number of state changes between \( t - j \) and \( t \) is even. This probability is given by

\[
prob[\pi_t = \pi_{t-j}] = \begin{cases} 
  j! (1 - \lambda)^j \sum_{n=0}^{j/2} \frac{(\lambda^2)^n((1-\lambda)^{-2})^n}{(j-2n)!((2n)!)}, & j \text{ even} \\
  j! (1 - \lambda)^j \sum_{n=0}^{(j-1)/2} \frac{(\lambda^2)^n((1-\lambda)^{-2})^n}{(j-2n)!((2n)!)}, & j \text{ odd}
\end{cases},
\]

which is derived in Appendix C. This probability converges towards \( 1/2 \) and, since \( \lambda < \frac{1}{2} \), it decreases monotonically in \( j \). This means that the longer the time since the last update, the lower the probability that income risk is still the same.

When in the period of the dynasty’s last update, \( t - j \), the state of the world was bad, the dynasty’s beliefs evolve according to

\[
p_{i,t}^h = prob[\pi_t = \pi_{t-j}]
\]

until the next update, with \( prob[\pi_t = \pi_{t-j}] \) given by equation (30). In case the state of the world was good in \( t - j \), beliefs evolve as

\[
p_{i,t}^h = 1 - prob[\pi_t = \pi_{t-j}]
\]

until the next update. Beliefs thus converge (from above or below) towards \( 1/2 \). The speed of convergence is the same for both, equations (31) and (32). Since \( p_{i,t}^h = 1/2 \) is always in the updating range if such range exists, beliefs reach the updating range in both political regimes.
Note that explicit updating is not the only source of complete information about income risk. Since agents vote truthfully, the outcome of the referendum in period $t$ is a perfect signal about what agents who updated in period $t - 1$ observed. When the agent observes an unexpected change in the result of the election, this can only be due to the fact that some agents have observed a change in the state of the world. This signal is observable for all agents and agents’ beliefs will thus be identical afterwards. This way, the updating decision will be perfectly synchronized across the population. As a consequence, all agents within one generation have identical prior beliefs, $p_{i,t}^h = p_i^h \forall i$. Since the prior belief determines all decisions of an agent, also all decisions are taken in an identical way by all agents within one generation, $\tau_{i,t} = \tau_t$, $d_{i,t} = d_t$, $p_{i,t+}^h = p_{t+}^h$, $s_{i,t} = s_t \forall i$.

4 Aggregate Dynamics

In this section, I describe the dynamics of the model. First, I will develop two important concepts for the dynamics of the model, the duration of inattentiveness and the political delay. Then, I will present the responses to a change in the fundamental income risk. I will also explain the dynamics of the model for an example where no shifts in income risk occur.

4.1 Duration of inattentiveness and political delay

The duration of inattentiveness $I(\tau)$ is the time between two updates and depends on the current political regime described by $\tau$. This time is only finite when, for some prior belief $p_i^h$, agents decide to update information or, technically, when an updating range exists for the current political regime. If an updating range exists, the duration of inattentiveness can be determined as follows. After an updating period $t - j$, agents’ beliefs move into the direction of the updating range according to equations (31) or (32). The speed of this movement is independent of the state of the world in the previous updating period. In addition the distance to the updating range is independent of the state of the world in the previous updating period since this range is symmetric around 1/2, i.e. $\bar{p}^0 = 1 - \underline{p}^0$ and $\bar{p}^1 = 1 - \underline{p}^1$. However, the distance to the updating range does depend on the current political regime since $\underline{p}^0 < \bar{p}^1$.

In the absence of social insurance, the duration of inattentiveness
\( I (0) \) is the time between the last update and the first period in which prior beliefs are within \((\overline{p}^0, 1 - \overline{p}^0)\),

\[
I (0) = \min \{ t \in \mathbb{N} \mid \text{prob} [\pi_t = \pi_{t-j}] < 1 - \overline{p}^0 \}.
\]

Analogously, in the presence of social insurance, the duration of inattentiveness is

\[
I (1) = \min \{ t \in \mathbb{N} \mid \text{prob} [\pi_t = \pi_{t-j}] < 1 - \overline{p}^1 \}.
\]

Since, if \( \overline{p}^0 \) and \( \overline{p}^1 \) exist, it holds that \( \overline{p}^0 < \overline{p}^1 \), the duration of inattentiveness is never longer without social insurance than with social insurance,

\[
I (0) \leq I (1).
\]

The political delay is the time between a change in the fundamental income risk and the implementation of the appropriate policy reform. The notion of a political delay implies that a certain policy reform is actually caused by a change in fundamentals. This is ensured when the expected indirect utility functions \( V \) and \( W \) intersect in the updating ranges. Then, a policy reform only takes place when agents actually observe that the true current state of the world is different from the state revealed by their last update.

The political delay then depends on the duration of inattentiveness and the timing of the change in the fundamental. The maximum delay is the duration of inattentiveness \( I \) and occurs when the change in the fundamental happens right after agents have updated. Due to the timing of events, the minimum delay is one period and occurs when income risk changes right before agents’ next update. Since state changes occur with equal probability each period, all delays between the minimum and maximum delay are equally likely. The expected political delay is thus

\[
D (\tau) = \frac{1}{2} \cdot (I (\tau) + 1),
\]

where \( \tau \) indicates the initial political regime. Since \( I (0) \leq I (1) \), the expected political delay is never longer in the absence of social insurance than in the presence of it,

\[
D (0) \leq D (1).
\]

This result relies on the disincentive effects of social insurance. In the presence of this welfare-state measure, agents save less and can thus gain less from information. As a consequence, agents remain inattentive for longer periods of time. Changes in income risk are then, in expectations, realized later and reforms have longer delays.

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4.2 Welfare-state dynamics

In the following, I present three experiments to illustrate the dynamics of the model. In all experiments, I consider a constellation where the duration of inattentiveness is finite in both political regimes. Furthermore, the expected indirect utility functions $V$ and $W$ intersect in their updating ranges such that reforms only take place after actual changes in income risk. I consider a case where agents prefer social insurance for low levels of income risk. As discussed before, the duration of inattentiveness and the expected political delay are not affected by this assumption. An example for a parameter constellation yielding the above is $\pi^l = 0.5$, $\pi^h = 0.9$, $e = 0.92$, $\lambda = 0.1$, $\kappa = 0.001$.

In this parameter constellation, $V(0) < W(0)$ and $V(1) > W(1)$. The updating ranges are given by $p^0 = 0.1471$, $\bar{p}^0 = 0.8529$, $p^1 = 0.1802$ and $\bar{p}^1 = 0.8198$. $V$ and $W$ intersect at $p^* = 0.3095$ and thus in the updating ranges. The duration of inattentiveness is $I(0) = 2$ in the absence of social insurance and $I(1) = 3$ in the presence of social insurance. Figures 5 to 7 are qualitative sketches of the dynamics under this parameter constellation. The general insights regarding the duration of inattentiveness and political delays are also valid for other constellations of $\pi^l$, $\pi^h$, $e$ and $\lambda$ together with $\kappa$ low enough for updating ranges to exist. For illustrational purposes, the sketches magnify certain areas while other areas are scaled down. The exact coordinates of belief-utility combinations sketched in the figures can be found in Appendix D.

In the first experiment, I consider a scenario where income risk is constant and, consequently, no policy reform takes place. The second experiment describes the model dynamics in a scenario where a change in income risk justifies the implementation of social insurance whereas the third experiment deals with the removal of this welfare-state measure.

**Scenario 1: Dynamics without changes in income risk.** Consider a scenario where no state change realizes for a certain time such that income risk is constant at its higher level $\pi^h$, i.e. $\pi_1 = \pi_2 = \ldots = \pi^t$. For this scenario, the dynamics of the model are illustrated in Figure 5.

I begin the description of the dynamics in a period 1 where, for their prior belief $p^h_1$, agents find it optimal to update. Due to the information update, agents know for sure that the state of the world is bad, i.e. that $\pi_1 = \pi^h$. Their posterior belief is therefore $p^h_{1+} = 1$. This situation corresponds to the point labeled "1+" in Figure 5.
Figure 5: Model dynamics without a change in income risk ($\pi_1 = \pi_2 = \pi_3 = \pi^h$).

One period later, the state of the world has become uncertain since a state change may have taken place between periods 1 and 2. According to equation (31), the probability assigned to the bad state of the world is now lower, $p_2^h < 1$, due to the possibility of state change, $\lambda > 0$. In Figure 4, agents still prefer no social insurance, $V (p_2^h) > W (p_2^h)$, and vote accordingly. Furthermore, the information that $\pi_1 = \pi^h$ is still worth that much that agents find it optimal not yet to update since $p_2^h > \bar{p}^0$. Thus, prior and posterior beliefs are identical in the second period, $p_2^h = p_2^h$. This situation corresponds to the point "2" in the figure.

In the third period, the value of the information that $\pi_1 = \pi^h$ has deteriorated further such that $p_3^h < p_2^h$ (point "3" in the figure). However, agents still find it optimal to have no social insurance, $V (p_3^h) > W (p_3^h)$. But agents are now so uncertain about income risk that they find it optimal to update since $p_3^h < \bar{p}^0$. Agents thus get informed about the true state of the world where income risk is still $\pi_3 = \pi^h$. Correspondingly,
their posterior belief in period 3 is $p_{3+}^h = 1$ as in period 1.

Period 4 then begins with the same prior belief as period 2, $p_4^h = p_2^h$. Since behavior within periods is solely determined by the prior belief, all decisions in period 4 are identical to the ones in period 2. By the same logic, period 5 is identical to period 3 and so on.

**Scenario 2: An implementation of social insurance.** I will now present the dynamics of the model after a change in income risk which makes a social insurance beneficial to agents. I will again begin with a period "1" in which prior beliefs are such that agents decide to update. The change in income risk happens between periods 1 and 2 and income risk is constant afterwards, $\pi_1 = \pi^h$, $\pi_2 = \pi_3 = ... = \pi^l$. Suppose there is no social insurance in period 1, where it is socially suboptimal. The dynamics of the model in this scenario are illustrated in Figure 6.

Behavior in periods 1 and 2 is as in the previous experiment. In period 2, the state of the world is different than in the previous experiment
but agents find it optimal not to update and therefore do not notice the change in fundamentals.

In period 3, the prior belief $p_{3}^{h}$ is such that agents still vote against social insurance but now find it optimal to update. Agents thus observe the true state of the world and notice that it has changed since their last update, $\pi_{3} = \pi^{l}$. Accordingly, the posterior belief in this period is $p_{3}^{h+} = 0$. Agents would now prefer social insurance, $W(0) > V(0)$, but have already decided against its implementation.

At the beginning of period 4, the information that $\pi_{3} = \pi^{l}$ has lost in value since it is possible that income risk has changed again between periods 3 and 4, therefore $p_{4}^{h} > 0$. However, agents still believe income risk to be rather low due to $\lambda < 1/2$. Consequently, they prefer social insurance in this period, $W(p_{4}^{h}) > V(p_{4}^{h})$, and implement it, $\tau_{4} = 1$.

In this example, social insurance was implemented with a delay of 2 periods (periods 2 and 3) after the change in fundamentals which justified the implementation. This absolute delay of the implementation depends on the parameter constellation chosen and is thus rather uninformative. It is more informative to consider the relative delay compared to a scenario where income risk shifts into the opposite direction. In the next section, I will present this experiment using the same parameter constellation underlying Figures 5 to 7.

**Scenario 3: A removal of social insurance.** This section presents the dynamics of the model when income risk changes into the opposite direction than in the previous experiment, i.e. I consider a change from low to high income risk between periods 1 and 2. After this single state change, income risk is constant, so that $\pi_{1} = \pi^{l}$, $\pi_{2} = \pi_{3} = ... = \pi^{h}$. Suppose there is social insurance in period 1, where it is socially optimal. Figure 7 illustrates the dynamics of the model in this experiment.

Suppose again that, in period 1, prior beliefs are such that agents find updating optimal. Agents thus observe that the state of the world is good, i.e. $\pi_{1} = \pi^{l}$. Since this piece of information is sure, agents’ posterior beliefs in period 1 are characterized by $p_{1+}^{h} = 0$.

At the beginning of period 2, agents are aware of the possibility that the state of the world could have changed between periods 1 and 2. Consequently, they assign a positive (low) probability to the bad state of the world, $p_{2}^{h} > 0$. However, agents still prefer social insurance, $W(p_{2}^{h}) > V(p_{2}^{h})$. Furthermore, beliefs are still certain enough such that
agents decide against updating their beliefs and thus do not notice the state change that occurred between periods 1 and 2.

At the beginning of period 3, the information \( \pi_1 = \pi^l \) has further deteriorated in value, \( p^h_3 > p^h_2 \), but the social insurance is still believed to be beneficial given agents’ expectations and thus it is not removed. If there were no social insurance, agents would find it optimal to update their beliefs now, since \( p^h_3 > p^0 \). However, since there is social insurance, gains from updating are lower and agents decide not yet to update, \( p^h_3 < p^1 \).

Consequently, prior beliefs in period 4 still only reflect the possibility of a state change but not the fact that one state change has actually taken place. The probability agents assign to the bad state of the world has further increased, \( p^h_4 > p^h_3 \), but not sufficiently to induce a change in policy. However, beliefs are now uncertain enough to cause agents update their beliefs. Observing the true state of the world, agents now realize that it has changed since their last update, \( \pi_4 = \pi^h \). Thus
posterior beliefs are now $p_{4+}^h = 1$. Agents would now like to change the political regime but can not do so before the next period.

In period 5, agents eventually change the political regime and remove social insurance which is no longer optimal. This political reform is implemented with a delay of 3 periods (periods 2, 3, and 4) after the change in fundamentals which justifies it. Again, this number alone is not very informative. But compared to the delay of 2 periods in the opposite experiment discussed previously, this model evaluation demonstrates that it takes longer to remove social insurance than to implement it.

The reason for this asymmetry is that informational incentives differ across political regimes. In the absence of social insurance, agents save more and update their information more frequently, in this example every 2 periods. When there is social insurance, private savings are crowded out and the duration of inattentiveness is longer (3 periods in this example). Without social insurance, changes in fundamentals are thus realized and political reforms carried out after two periods at the latest, while this can take three periods in the presence of social insurance.

### 4.3 Numerical Evaluation

In this section, I evaluate the duration of inattentiveness and the expected political delay after changes in fundamentals numerically. Specifically, I analyze how the political delay depends on the information cost $\kappa$. It is clear from the theoretical considerations that higher updating costs make agents less attentive and increase both, the duration of inattentiveness and the political delay. However, it is not possible to quantify this effect analytically. This section present a numerical quantification of this effect. Furthermore, the numerical evaluation allows to analyze how the size of the information cost affects the relative duration of inattentiveness in the two political regimes, $I (1) / I (0)$.

In order to highlight the role of the information cost $\kappa$, I present results for different values of $\kappa$ holding constant the other parameters of the model. Specifically, I consider the constellation $\pi^l = 0.25$, $\pi^h = 0.75$, and $\epsilon = 0.933$. These parameter values imply that the indirect utility functions $V$ and $W$ intersect at $p^* = 0.5$. Thus political reforms are only implemented after agents have updated beliefs. Furthermore, I set $\lambda = 0.1$ implying that the expected duration of a state of the world is
<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\kappa/\tilde{V}(0)$</th>
<th>cons. equ.</th>
<th>$I(0)$</th>
<th>$I(1)$</th>
<th>$D(0)$</th>
<th>$D(1)$</th>
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<td>0.0053</td>
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<tr>
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<td>1.5</td>
</tr>
<tr>
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<td>2</td>
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<td>1.5</td>
</tr>
<tr>
<td>0.0211</td>
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<td>0.59%</td>
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<td>4</td>
<td>2.0</td>
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</tr>
<tr>
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<td>0.73%</td>
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<td>8</td>
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<td>4.5</td>
</tr>
<tr>
<td>0.0272</td>
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<td>0.76%</td>
<td>5</td>
<td>21</td>
<td>3.0</td>
<td>11.0</td>
</tr>
<tr>
<td>0.0290</td>
<td>0.55%</td>
<td>0.81%</td>
<td>6</td>
<td>$\infty$</td>
<td>3.5</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.0317</td>
<td>0.60%</td>
<td>0.89%</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 1: Duration of Inattentiveness and Expected Political Delay for Different Informations Costs $\kappa$ ($\pi^l = 0.25$, $\pi^h = 0.75$, $e = 0.933$, $\lambda = 0.1$)

ten periods. Table 1 presents the duration of inattentiveness and the expected political delay for different values of the information cost $\kappa$.

In order to put the absolute level of the information cost $\kappa$ into perspective, the table also reports $\kappa$ relative to full-information lifetime utility in the good state, $\kappa/\tilde{V}(0)$. The third column expresses the utility cost in terms of consumption, the reported numbers are the relative reductions of income that would result in a utility loss of $\kappa$ in the good state of the world without social insurance. The fourth column of the table reports the time between two updates in the absence of social insurance, $I(0)$, and the fifth column reports the duration of inattentiveness when there is social insurance, $I(1)$. The last two columns report the expected duration $D(\tau)$ between a change in income risk and the appropriate change in policy in the two political regimes, where $\tau$ indicates the initial political regime. $D(0)$ is the expected delay of an implementation of social insurance and $D(1)$ is the expected delay of a removal of social insurance.

In the first row of Table 1, information costs are rather small and amount to only 0.1% of lifetime utility. In this setting, agents find it optimal to update their beliefs in every period in both political regimes. Thus the time between two updates is 1. Consequently, we also observe the minimum political delay of one period between a change in
fundamentals and the implementation of the appropriate policy reform.

With higher information costs of 0.2% of expected lifetime utility (second row of the table), agents still find it rational to update every period when there is no social insurance. However, in the presence of social insurance, gains from updating are lower and agents only update every second period. Consequently, a change in income risk justifying the implementation of social insurance is translated into a policy reform right in the next period. By contrast, removals of social insurance can have a delay of two periods.

Further increases in the information cost leads to longer durations of inattentiveness and, in consequence, to longer political delays. Since gains from updating are always lower in the presence of social insurance, the duration of inattentiveness and expected political delays are longer in this political regime.

The next to last row of Table 1 presents a scenario where information costs (0.029) are such that, without social insurance, agents find it optimal to update their beliefs every six periods but never update in the presence of social insurance. In this case, condition (29) is not fulfilled for any $p_{i,t} \in [0, 1]$. In this scenario, the society implements social insurance with an expected delay of 3.5 periods. Once this political regime is implemented, agents decide to be inattentive forever and thus the social insurance will never be removed independent of the underlying state of the world. Thus welfare-state persistence is eternal in this scenario.

The same holds in the last row of Table 1 where the information cost is set to 0.0317. Here, agents are also completely inattentive in the absence of social insurance. Political reforms thus never take place. The economy remains in its initial political regime forever.

Note that relatively low values for the information cost are sufficient to generate these extreme forms of political persistence. In the scenarios displayed in the last two rows of Table 1, information costs amount to 0.55% and 0.6% of expected lifetime utility under full information in the good state of the world, respectively, which is equivalent to a loss of less than 1% of consumption. Reis (2006a) discusses different parametrization of his inattentiveness model with updating costs ranging from 0.2% to 0.8% of income. Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) measure updating and planning costs of a firm and find that these costs are roughly 1% of total revenue.
5 Conclusion

This paper has offered an information-based explanation for welfare-state persistence. The explanation is based on the effects of the welfare state on attentiveness. I argued that the welfare state crowds out private financial precautions and this way reduces incentives to be attentive to developments in economic fundamentals. Knowledge about fundamentals does not only influence private decisions on savings or insurance but is also important for optimal political choices.

When people face a cost of processing information, they will choose how often to inform themselves about fundamentals. The incentives to do so depend on their level of private financial precaution. In turn, the incentives for financial precaution depend on welfare-state arrangements. For instance, if the degree of social insurance is high, people engage little in private financial activity. Consequently, they remain inattentive to news for longer periods. As a result, it takes long until a change in fundamentals is noticed by a majority of society and translated into appropriate policies if initial welfare-state arrangements are pronounced.

I have presented a model where rationally inattentive agents face an unknown degree of income risk. Agents decide on private savings, attentiveness to news and whether to vote in favor of social insurance. When society has implemented social insurance, agents save less and are, consequently, less attentive to news since choosing savings based on better information has a lower impact on lifetime utility. This way, welfare-state persistence arises from the incentive effects of the welfare state on attentiveness.

References


Ball, L., G. N. Mankiw, and R. Reis (2005). Monetary policy for inat-


**Appendix**

**A  Savings decision**

**A.1  Optimal savings when \( \tau_t = 0 \)**

Under \( \tau_t = 0 \), \( x_{i,t,t+1} \) can take only two values: \( x^{L,t} = x^{h,L} = x^L = 1 \) and \( x^{h,U} = x^{l,U} = x^U = 0 \). From the perspective of agent \((i,t)\) after the updating decision, the subjective probability of these two outcomes are

\[
prob_{i,t+}[x_{i,t,t+1} = x^L] = prob_{i,t+}[x_{i,t,t+1} = x^{h,L}] + prob_{i,t+}[x_{i,t,t+1} = x^{l,L}]
\]

\[
= prob_{i,t+}[y_{i,t,t+1} = 1 \mid \pi_t = \pi^h] \cdot prob_{i,t+}[\pi_t = \pi^h]
\]

\[
+ prob_{i,t+}[y_{i,t,t+1} = 1 \mid \pi_t = \pi^l] \cdot prob_{i,t+}[\pi_t = \pi^l]
\]

\[
= (1 - \pi^h) \cdot p^h_{i,t+} + (1 - \pi^l) \cdot (1 - p^l_{i,t+})
\]

\[
= 1 - \pi^e_{i,t+}
\]

and

\[
prob_{i,t+}[x_{i,t,t+1} = x^U] = \pi^e_{i,t+}.
\]

Hence, the consumption Euler equation reads as

\[
\frac{u'(1 - s_{i,t})}{u'(1 + s_{i,t}) + \pi^e_{i,t+}} \cdot u'(s_{i,t}).
\]

Using the functional form of marginal utility, \( u'(c_{i,t,t+h}) = 4 - 2c_{i,t,t+h} \), \( h = 0, 1 \), gives

\[
4 - 2(1 - s_{i,t}) = (1 - \pi^e_{i,t+}) \cdot [4 - 2(1 + s_{i,t})] + \pi^e_{i,t+} \cdot [4 - 2(s_{i,t})].
\]

This equation is solved by

\[
s_{i,t} = \frac{\pi^e_{i,t+}}{2},
\]

which is the expression for optimal savings in the absence of social insurance stated in equation (20).
A.2 Optimal savings when $\tau_t = 1$

Under $\tau_t = 1$, $x_{i,t,t+1}$ can take only two values: $x^{l,L} = x^{l,U} = x^l = (1 - \pi^l) e$ and $x^{h,L} = x^{h,U} = x^h = (1 - \pi^h) e$. From the perspective of agent $(i,t)$ after the updating decision, the subjective probability of these two outcomes are

$$prob_{i,t+} \left[ x_{i,t,t+1} = x^l \right] = 1 - p^h_{i,t+},$$

and

$$prob_{i,t+} \left[ x_{i,t,t+1} = x^h \right] = p^h_{i,t+}.$$

Hence, the consumption Euler equation reads as

$$u' \left( e - s_{i,t} \right) = \left( 1 - p^h_{i,t+} \right) \cdot u' \left( (1 - \pi^l) e + s_{i,t} \right) + p^h_{i,t+} \cdot u' \left( (1 - \pi^h) e + s_{i,t} \right).$$

Using the functional form of marginal utility, $u' \left( c_{i,t,t+h} \right) = 4 - 2c_{i,t,t+h}$, $h = 0, 1$, gives

$$4 - 2 \left( e - s_{i,t} \right) = \left( 1 - p^h_{i,t+} \right) \cdot \left( 4 - 2 \left[ (1 - \pi^l) e + s_{i,t} \right] \right)$$

$$+ p^h_{i,t+} \cdot \left( 4 - 2 \left[ (1 - \pi^h) e + s_{i,t} \right] \right)$$

$$\iff 2s_{i,t} = \left[ \left( 1 - p^h_{i,t+} \right) \cdot \pi^l + p^h_{i,t+} \cdot \pi^h \right] e.$$

This can be simplified to

$$s_{i,t} = \frac{\pi_{i,t+}^e e}{2},$$

which is equation (21).

B Expected indirect utility

B.1 Expected indirect utility when $\tau_t = 0$

For a savings level $s_{i,t}$, consumption in period $t$ and in the two possible realizations of net income in period $t + 1$ are $c_{i,t,t} = 1 - s_{i,t}$, $c_{i,t,t+1}^L = 1 + s_{i,t}$, and $c_{i,t,t+1}^U = s_{i,t}$ when $\tau_t = 0$. In the absence of social insurance, expected lifetime utility net of updating costs is thus

$$\bar{V} := E_{i,t+} \bar{U}_{i,t} = u \left( 1 - s_{i,t} \right) + \left( 1 - \pi_{i,t+}^e \right) \cdot u \left( 1 + s_{i,t} \right) + \pi_{i,t+}^e \cdot u \left( s_{i,t} \right),$$

which, for (2), becomes

$$\bar{V} = 4 + 4 \left( 1 - \pi_{i,t+}^e \right) - \left( 1 - s_{i,t} \right)^2 - \left( 1 - \pi_{i,t+}^e \right) \cdot \left( 1 + s_{i,t} \right)^2 - \pi_{i,t+}^e \cdot \left( s_{i,t} \right)^2.$$
Using the optimal savings (20) gives

\[ \tilde{V} = 4 + 4 \left( 1 - \pi_{i,t+}^e \right) - \left( 1 - \frac{\pi_{i,t+}^e}{2} \right)^2 - \left( 1 - \frac{\pi_{i,t}^e}{2} \right)^2 - \pi_{i,t}^e \left( \frac{\pi_{i,t+}^e}{2} \right)^2. \]

This evaluates as

\[ \tilde{V} = 6 - 3\pi_{i,t+}^e + \frac{(\pi_{i,t+}^e)^2}{2}, \]

which is equation (22). To derive the marginal derivatives of \( \tilde{V} \) with respect to \( p_{i,t+}^h \), first take the derivatives with respect to perceived income risk \( \pi_{i,t+}^e \):

\[ \frac{\partial \tilde{V}}{\partial \pi_{i,t+}^e} = -3 + \pi_{i,t+}^e < 0 \text{ since } \pi_{i,t}^e \leq 1, \]

\[ \frac{\partial^2 \tilde{V}}{\partial (\pi_{i,t+}^e)^2} = 1 > 0. \]

\( \tilde{V} \) is thus decreasing and convex in \( \pi_{i,t+}^e \). As \( \pi_{i,t+}^e = \pi^e + p_{i,t+}^h (\pi^h - \pi^l) \) is a linear and increasing function of \( p_{i,t+}^h \), \( \tilde{V} \) is also decreasing and convex in \( p_{i,t+}^h \):

\[ \frac{\partial \tilde{V}}{\partial p_{i,t+}^h} = \frac{\partial \tilde{V}}{\partial \pi_{i,t+}^e} \cdot \frac{\partial \pi_{i,t+}^e}{\partial p_{i,t+}^h} = (-3 + \pi_{i,t+}^e) \cdot (\pi^h - \pi^l) < 0, \]

\[ \frac{\partial^2 \tilde{V}}{\partial (p_{i,t+}^h)^2} = (\pi^h - \pi^l)^2 > 0. \]

**B.2 Expected indirect utility when \( \tau_t = 1 \)**

For a savings level \( s_{i,t} \), consumption in period \( t \) and in the two possible realizations of net income in period \( t + 1 \) are \( c_{i,t,t} = e - s_{i,t} \), \( c_{i,t,t+1} = (1 - \pi^l) e + s_{i,t} \), and \( c_{i,t,t+1}^h = (1 - \pi^h) e + s_{i,t} \) when \( \tau_t = 1 \). In the presence of social insurance, expected lifetime utility net of updating costs is thus

\[ \tilde{W} : = E_{i,t+} \tilde{U}_{i,t} = u(e - s_{i,t}) + (1 - p_{i,t+}^h) \cdot u ((1 - \pi^l) e + s_{i,t}) + p_{i,t+}^h \cdot u ((1 - \pi^h) e + s_{i,t}). \]
which, for (2), becomes

\[
\tilde{W} = 4e - (e - s_{i,t})^2 + 4e \left[ (1 - p_{i,t,t+}^h) \cdot (1 - \pi^l) + p_{i,t,t+}^h \cdot (1 - \pi^h) \right] \\
- (1 - p_{i,t,t+}^h) \cdot \left( (1 - \pi^l) e + s_{i,t} \right)^2 - p_{i,t,t+}^h \cdot \left( (1 - \pi^h) e + s_{i,t} \right)^2 \\
= 8e - (e - s_{i,t})^2 - 4\pi_i e - (1 - p_{i,t,t+}^h) \cdot \left( (1 - \pi^l) e + \frac{\pi_i}{2} e \right)^2 \\
- p_{i,t,t+}^h \cdot \left( (1 - \pi^h) e + \frac{\pi_i}{2} e \right)^2 ,
\]

Using the optimal savings (21) gives

\[
\tilde{W} = 8e - \left( e - \frac{\pi_i^e}{2} e \right)^2 - 4\pi_i e - (1 - p_{i,t,t+}^h) \cdot \left( (1 - \pi^l) e + \frac{\pi_i^e}{2} e \right)^2 \\
- p_{i,t,t+}^h \cdot \left( (1 - \pi^h) e + \frac{\pi_i^e}{2} e \right)^2 ,
\]

which can be simplified to

\[
\tilde{W} = 8e - (1 - \pi_i^e) e^2 - \frac{(\pi_i^e)^2 e^2}{2} - 4e\pi_i^e \\
- 2E_{i,t} \left( 1 - \pi_t \right)^2 - \frac{(\pi_i^e)^2 e^2}{2} - 4\pi_i^e e \\
\]

which is the expression in equation (23). The marginal derivatives with respect to \( p_{i,t,t+}^h \) are

\[
\frac{\partial \tilde{W}}{\partial p_{i,t,t+}^h} = (-2e + \pi_i^e) \cdot \frac{\partial \pi_i^e}{\partial p_{i,t,t+}^h} - e^2 E_{i,t} \left( \pi_t \right)^2 \\
= (-2e + \pi_i^e) \cdot (\pi^h - \pi^l) - e^2 \left( (\pi^h)^2 - (\pi^l)^2 \right) ,
\]
which is negative since \( e < 1 \), \( \pi_{i,t}^e \leq 1 \), and \( \pi^h > \pi^l \), and

\[
\frac{\partial^2 \tilde{W}}{\partial (p^h_{i,t+})^2} = e^2 \cdot (\pi^h - \pi^l)^2 > 0.
\]

\( \tilde{W} \) is thus decreasing and convex in \( p^h_{i,t+} \).

### B.3 Intersection of \( V \) and \( W \)

In this appendix, I present two examples of parameter constellations where the expected indirect utility functions \( V \) and \( W \) intersect on \((0, 1)\). \( V \) cuts \( W \) from below in one example and from above in the other example. Furthermore, in these examples, \( V \) and \( W \) intersect at \( p^* = 0.5 \) which is always in the updating ranges (if they exist).

Consider first the parameter constellation \( \pi^l = 0.7749 \), \( \pi^h = 0.9589 \), and \( e = 0.9355 \). In this constellation, \( \tilde{V}(0) \), \( \tilde{V}(1) \), \( \tilde{W}(0) \) and \( \tilde{W}(1) \) evaluate as \( \tilde{V}(0) = 3.9755 \), \( \tilde{V}(1) = 3.5830 \), \( \tilde{W}(0) = 4.0212 \), and \( \tilde{W}(1) = 3.5373 \). I.e. it holds that \( \tilde{V}(0) < \tilde{W}(0) \) and \( \tilde{V}(1) > \tilde{W}(1) \). Thus \( V \) cuts \( W \) from below in this example.

Now consider the parameter constellation \( \pi^l = 0.25 \), \( \pi^h = 0.75 \), and \( e = 0.9337 \). In this constellation, \( \tilde{V}(0) \), \( \tilde{V}(1) \), \( \tilde{W}(0) \) and \( \tilde{W}(1) \) evaluate as \( \tilde{V}(0) = 5.2813 \), \( \tilde{V}(1) = 4.0313 \), \( \tilde{W}(0) = 5.2321 \), and \( \tilde{W}(1) = 4.0804 \). I.e. it holds that \( \tilde{V}(0) > \tilde{W}(0) \) and \( \tilde{V}(1) < \tilde{W}(1) \). Thus, in this example, \( V \) cuts \( W \) from above.

If \( V \) and \( W \) intersect in their updating ranges, then this intersection is at

\[
p^h_{i,t} = \frac{\tilde{V}(0) - \tilde{W}(0)}{\tilde{V}(0) - \tilde{W}(0) - \tilde{V}(1) + \tilde{W}(1)}.
\]

In both examples above this expression evaluates as \( p^h_{i,t} = 0.5 \). If an updating range exists, \( p^h_{i,t} = 0.5 \) is in this range. Provided that \( \kappa \) is small enough that \( V \) and \( W \) have updating ranges, the two functions intersect in their updating ranges in both examples.

### C Belief formation

The probability that \( \pi \) is the same as \( j \) periods ago, i.e. \( \pi_t = \pi_{t-j} \), is the probability that the number of regime shifts between \( t-j \) and \( t \) is even. Using properties of the binomial distribution, this probability can
be calculated as

\[ p[\pi_t = \pi_{t-j}] = \binom{j}{0} \cdot \lambda^0 (1 - \lambda)^{j-0} + \binom{j}{2} \cdot \lambda^2 (1 - \lambda)^{j-2} + \binom{j}{4} \cdot \lambda^4 (1 - \lambda)^{j-4} + \ldots \]

\[ = \begin{cases} 
\sum_{n=0}^{j/2} \binom{2n}{j} \cdot \lambda^{2n} (1 - \lambda)^{j-2n}, & \text{\(j\) even} \\
\sum_{n=0}^{(j-1)/2} \binom{2n}{j} \cdot \lambda^{2n} (1 - \lambda)^{j-2n}, & \text{\(j\) odd} 
\end{cases} \]

For the case of \(j\) being even, this probability can be simplified as follows:

\[ \sum_{n=0}^{j/2} \binom{j}{2n} \cdot \lambda^{2n} (1 - \lambda)^{j-2n} = \sum_{n=0}^{j/2} \frac{j!}{(j-2n)! (2n)!} \cdot (\lambda^2)^n (1 - \lambda)^{j} (1 - \lambda)^{-2n} \]

\[ = j! (1 - \lambda)^j \sum_{n=0}^{j/2} \frac{(\lambda^2)^n ((1 - \lambda)^{-2})^n}{(j-2n)! (2n)!} \]

Analogously, for the case that \(j\) is odd, the probability can be simplified to:

\[ \sum_{n=0}^{(j-1)/2} \binom{j}{2n} \cdot \lambda^{2n} (1 - \lambda)^{j-2n} = j! (1 - \lambda)^j \sum_{n=0}^{(j-1)/2} \frac{(\lambda^2)^n ((1 - \lambda)^{-2})^n}{(j-2n)! (2n)!} \]

such that

\[ p[\pi_t = \pi_{t-j}] = \begin{cases} 
j! (1 - \lambda)^j \sum_{n=0}^{j/2} \frac{(\lambda^2)^n ((1 - \lambda)^{-2})^n}{(j-2n)! (2n)!}, & \text{\(j\) even} \\
j! (1 - \lambda)^j \sum_{n=0}^{(j-1)/2} \frac{(\lambda^2)^n ((1 - \lambda)^{-2})^n}{(j-2n)! (2n)!}, & \text{\(j\) odd} 
\end{cases} \]

which is equation (30).

### D Welfare-state dynamics

This Appendix presents the exact probabilities representing agents’ beliefs and the exact associated values for expected indirect utility in the three scenarios in Section 4.2. The values can be found in Table 2.
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<th>V</th>
<th>W</th>
<th>belief</th>
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<th>W</th>
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<th>W</th>
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<td>4.625</td>
<td>4.641</td>
<td>0.180</td>
<td>4.449</td>
<td>4.456</td>
</tr>
<tr>
<td>4</td>
<td>0.900</td>
<td>3.790</td>
<td>3.760</td>
<td>0.100</td>
<td>4.526</td>
<td>4.538</td>
<td>0.244</td>
<td>4.390</td>
<td>4.394</td>
</tr>
<tr>
<td>4+</td>
<td>0.900</td>
<td>3.790</td>
<td>3.760</td>
<td>0.100</td>
<td>4.526</td>
<td>4.538</td>
<td>0.900</td>
<td>3.790</td>
<td>3.760</td>
</tr>
<tr>
<td>5</td>
<td>0.820</td>
<td>3.861</td>
<td>3.834</td>
<td>0.180</td>
<td>4.449</td>
<td>4.456</td>
<td>0.820</td>
<td>3.861</td>
<td>3.834</td>
</tr>
</tbody>
</table>

Table 2: Dynamics of beliefs and expected indirect utility in the scenarios from Section 4.2 \((\pi^l = 0.5, \pi^h = 0.9, \epsilon = 0.92, \lambda = 0.1, \kappa = 0.001\); scenario 1: \(\pi_1 = \pi_2 = ... = \pi^h\), scenario 2: \(\pi_1 = \pi^h, \pi_2 = \pi_3 = ... = \pi^l\), scenario 3: \(\pi_1 = \pi^l, \pi_2 = \pi_3 = ... = \pi^h\).