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Abstract
We provide an axiomatic framework for exchange markets with a willingness-to-pay/willingness-to-accept discrepancy. First, we obtain a two parameter family of market invariants under price-scaling representing the excess demand. One of the parameters can be identified as endowment. The other is a new feature, called demand-supply gap, that leads to classical general equilibrium if zero. Second, we provide representations of price and demand as unbounded operators on an infinite dimensional Hilbert space. We prove that neither can this space be finite dimensional nor can these operators be bounded. Third, if the demand-supply gap is not zero we obtain that price and demand are not simultaneously sharply measurable and consequently a Walrasian equilibrium does not exist.

JEL Classification: D50, D51, D01, D03

Keywords: General equilibrium theory, endowment effect, non-existence of equilibrium

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1 Introduction

General equilibrium theory by Arrow, Debreu and McKenzie is nowadays a cornerstone of theoretical economics and present in various fields. However, general equilibrium has also been continuously challenged. From a theoretical perspective existence, uniqueness and stability of the competitive equilibrium are crucial. Arrow and Debreu (1954) secure the existence, Debreu (1970) studies the local uniqueness of the competitive equilibrium and Scarf (1967) provides an example that the tâtonnement price adjustment mechanism is not globally stable.

Critics [see, e.g., Blaug (1980) and Kirman (1989)] often argue that general equilibrium theory lacks empirical content. In particular they claim that testable implications of general equilibrium are missing. In this case Popperian falsification of general equilibrium theory is impossible. The common criticism is based on the aggregate excess demand function and the Sonnenschein-Mantel-Debreu theorem. Sonnenschein (1973), Mantel (1974) and Debreu (1974) show that the aggregate excess demand function can have arbitrary shapes as long as Walras' Law, continuity and homogeneity of degree zero in prices is satisfied.

New results show that falsification is possible and not all empirical outcomes can be rationalized by the equilibrium hypotheses. The seminal contribution of Brown and Matzkin (1996) is the first in a series of papers dealing with this issue. Building on Afriat (1967) they derive testable restrictions on the equilibrium manifold[1] introduced by Balasko (1975). Various extensions of the Brown and Matzkin framework have been developed by Kubler (2003) for expected utility, Snyder (1999) for public goods and Carvajal (2004) for random preferences [see also Carvajal, Ray and Snyder (2004), or Chiappori et al. (2004)].

For a long time market behavior and in particular the predictions of general equilibrium theory have also been a vivid field of research in experimental economics. One of the first studies is Chamberlin's (1948) market

[1]The equilibrium manifold is the set of price and endowment pairs with zero excess demand.

experiment in which prices and quantities failed to converge to the competitive equilibrium. In the following it was the seminal contribution of Smith (1962) to add a double-auction to Chamberlin's market environment and to show that now prices and quantities converge to the competitive equilibrium. Moreover, early experiments showed that an individual's valuation of goods crucially depends on whether the individual already owns or intends to buy the same good. Thaler (1980) coined the term *endowment effect* to account for the alleged tendency of individuals to state a higher minimum (in monetary units) for which the individual is willing to sell a good than the maximum the same individual is willing to pay to buy the same good. Various authors find support for the endowment effect [see, e.g., Knetsch (1989), Kahnemann, Knetsch and Thaler (1990), Kahnemann, Knetsch and Thaler (1991), Bateman et al. (1997) or Bauer and Schmidt (2008)] while others argue that the endowment effect is merely a result of inadequate experimental instructions [see Plott and Zeiler (2005)] or inexperienced agents [see, e.g., Shogren et al. (1994)]. Although this effect is still an active field of research [see, e.g., Horowitz and McConnell (2002) or List (2004)] the most accepted conjecture to explain this behavioral pattern is prospect theory by Kahnemann and Tversky (1979). The endowment effect has however, (to our knowledge) not been incorporated into a general equilibrium framework yet.

The aim of this paper is to introduce the possibility to model an endowment effect in a general equilibrium framework. In particular we are interested whether the endowment effect alters our understanding of classical general equilibrium results like the existence of the competitive equilibrium or Walrasian equilibrium prices. To do so, we consider an exchange market where the following main hypothesis mirrors the experimental evidence:

> *First selling and then buying a good does not necessarily lead to the same market state as first buying and then selling that good.*

The model is given as a set of axioms containing a real parameter called demand-supply gap. This parameter reflects the main hypothesis if we assume that an individual that first sells and than buys a good is endowed, while an individual that first buys and then sells the good is not endowed with the good. In our framework the demand-supply gap is empirically testable. The larger the modulus of the parameter the more prevalent is the endowment effect in the market. We distinguish two cases: If the demand-supply gap is zero (symmetric case) there exists no endowment effect and the model leads to classical general equilibrium. If the demand-supply gap is not equal to zero (asymmetric case) we obtain that the dispersion of price and the dispersion of demand in the same market state are both strictly larger than zero.

In general equilibrium theory it is assumed that demand is invariant under price scaling [see, e.g., Mas-Colell, Whinston and Green (1995, p. 23)]. In the asymmetric case such an assumption is not necessary. We derive a set of market invariants and show that the proposed axioms are unique to support an economically reasonable identification of these market invariants with demand and excess demand. We provide representations of price and demand as unbounded operators on an infinite dimensional Hilbert space. We prove that neither can this space be finite dimensional nor can these operators be bounded. Price and demand cannot be simultaneously sharply measured and as a consequence of this effect market clearing Walrasian equilibrium prices do not exist in the asymmetric case.

2 The Asymmetric Market Model

We assume $n \in \mathbb{N}$ distinguishable goods that are traded in a pure exchange market. The state of the market is given by a non-zero vector ξ in a Hilbert space X with inner product denoted by $\langle \cdot | \cdot \rangle$. Observables are self-adjoint operators on this Hilbert space. The markets we consider satisfy the following axioms:

(MA1) The price p_i of good i is a positive observable on X for all goods $1 \leq i \leq n$.

(MA2) The demand d_i of good i is an observable on X for all goods $1 \leq i \leq n$.

(MA3) The endowment ω_i of good i is a real number $\omega_i \in \mathbb{R}$ for all goods $1 \leq i \leq n$.

In our final axiom (and definition) we state the relation between price and demand.

(MA4) Prices p_i and demands d_j interact according to

$$[p_i, d_j] = i\mu_i p_i \delta_{i,j} \tag{1}$$

for a fixed real $\mu_i \in \mathbb{R}$ called the *demand-supply gap.*

Recall, that for observables a, b on X the commutator $[a, b]$ is defined as $[a, b] := ab - ba$ on the appropriate domain and the Kronecker symbol is defined as $\delta_{i,j} := \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$. A positive observable a on X is an observable with $\langle a\xi|\xi\rangle > 0$ for all $0 \neq \xi$ in the domain $D(a)$ of a. The vectors $(p_i)_{1\leq i\leq n}$, $(d_i)_{1\leq i\leq n}$ and $(\omega_i \mathrm{id}_X)_{1\leq i\leq n}$ are denoted by p, d and ω respectively. A market is called asymmetric if it satisfies the above four axioms with $\mu_i \neq 0$ for at least one good i.

With the fourth axiom we model that buying and selling of goods is exchangeable for different goods while if we buy and sell the same good, the difference of exchange is measured by the operator $i\mu_i p_i$. Let us digress for a moment and examine what possible right-hand sides $R(p_i)$ in (1) are reasonable for our investigations. Such a reasonable right-hand side must surely satisfy at least two conditions. First, it has to be positively homogeneous of degree one in p_i (i.e., $R(\mu p_i) = \mu R(p_i)$ for all $\mu \geq 0$) to secure independence of price scaling. Second, it has to be formally skew-adjoint (i.e. $\langle R(p_i)\xi|\zeta\rangle = \langle \xi| - R(p_i)\zeta\rangle$ on the appropriate domain) as can be seen as follows by considering the adjoint of equation (1): $R(p_i)^* = [p_i, d_j]^* = -[p_i, d_j] = -R(p_i)$. The operator $i\mu_i p_i$ is the simplest possible right-hand side to satisfy these two conditions and is therefore a natural choice for our purpose.

Measurement of an observable, e.g. the price of good i, in a market in state ξ (e.g. in this case selling a small quantity of good i) will result in a jump of the market into a new state ζ being an eigenvector of the observable.

The outcome of the measurement will be a real number ζ_i (e.g. the price), the eigenvalue of the observable corresponding to ζ with probability

$$\text{prob } (\zeta_i) = \frac{\langle \xi | \zeta \rangle \langle \zeta | \xi \rangle}{\|\xi\|^2}.$$

For an observable a on X one can show that its mean value at state $\xi \in X$ is given as

$$\overline{a_\xi} = \frac{\langle a\xi | \xi \rangle}{\|\xi\|^2}.$$

The dispersion of an observable a on X is given as

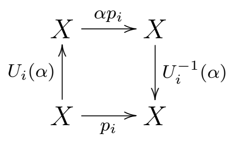
$$\overline{(\triangle a)^2_\xi} = \frac{\left\langle \left(a - \overline{a_\xi}\mathrm{id}_X\right)^2 \xi | \xi \right\rangle}{\|\xi\|^2}.$$

3 Market Invariants

The main purpose of this section is to derive the market invariants of the asymmetric market under price-scaling. For that purpose, let $(U_i(\alpha))_{0<\alpha\in\mathbb{R}}$ be a strongly continuous family of unitary operators on X such that

$$U_i^{-1}(\alpha) p_i U_i(\alpha) = \alpha p_i,$$

i.e., the following diagram commutes



The family $U_i(\cdot)$ satisfies the following properties for all $\alpha > 0$ and $\beta > 0$:

- $U_i(1) = \mathrm{id}_X$
- $U_i(\alpha)U_i(\beta) = U_i(\alpha\beta) = U_i(\beta)U_i(\alpha)$
- $U_i^{-1}(\alpha) = U_i\left(\frac{1}{\alpha}\right)$

Define $T_i(t) := U_i(e^t)$ and observe

- $T_i(0) = \mathrm{id}_X$
- $T_i(t)T_i(s) = T_i(t+s) = T_i(s)T_i(t)$
- $T_i^{-1}(t) = T_i(-t)$

This yields T_i to be a strongly continuous group of unitary operators acting on X. Thus, the theorem of Stone [see e.g. Engel and Nagel (2000)] ensures the existence of a skew-adjoint generator A_i. Set $\alpha = e^t$ and with $U(\alpha) = T(\ln \alpha)$ it follows that

$$\begin{aligned} p_i &= \frac{d}{d\alpha}\left(U_i^{-1}(\alpha) p_i U_i(\alpha)\right) \\ &= \frac{d}{d\alpha}\left(T_i(-\ln \alpha) p_i T_i(\ln \alpha)\right) \\ &= -\frac{1}{\alpha} T_i(-\ln \alpha) A_i p_i T_i(\ln \alpha) + \frac{1}{\alpha} T_i(-\ln \alpha) p_i A_i T_i(\ln \alpha). \end{aligned}$$

Evaluation at $\alpha = 1$ yields

$$[p_i, A_i] = p_i. \tag{2}$$

Since a generator commutes with the strongly continuous group it generates it is easily seen that $\beta_i A_i + \gamma_i \mathrm{id}_X$ also commutes with $U_i(\alpha)$ for any $\beta_i, \gamma_i \in \mathbb{C}$. Hence $\beta_i A_i + \gamma_i \mathrm{id}_X$ represents a market invariant under price-scaling. Before we give $\beta_i A_i + \gamma_i \mathrm{id}_X$ an economic meaning we further analyse A_i.

Lemma 1. Whenever $A_i \xi = \lambda \xi$, then $A_i p_i \xi = (\lambda - 1) p_i \xi$.

Proof. Since $p_i A_i - A_i p_i = p_i$ we can evaluate at a vector ξ and obtain $p_i A_i \xi - A_i p_i \xi = p_i \xi$. The assertion follows immediately. $\diamondsuit$

Since A_i is skew-adjoint its possible eigenvalues would be purely imaginary. The Lemma then implies that A_i does not have eigenvectors and eigenvalues. Moreover, the underlying Hilbert space X is infinite dimensional.

Lemma 2. For bounded linear operators A, B with $A^n \neq 0$ for all $n \in \mathbb{N}$ holds $[A, B] \neq A$.

Proof. Assume $[A, B] = A$ and as induction hypothesis $[A^n, B] = nA^n$. Then $[A^{n+1}, B] = A\,[A^n, B] + [A, B]\,A^n = nA^{n+1} + A^{n+1} = (n+1)A^{n+1}$. The norm estimate $n\,\|A^n\| = \|[A^n, B]\| \leq 2\,\|A^n\|\,\|B\|$ yields the desired contradiction since $A^n \neq 0$ for all $n \in \mathbb{N}$. ◇

Since p_i is positive the lemma is applicable and hence, at least one of the operators p_i and A_i is unbounded.

4 Economic Interpretation

Now we derive an economic interpretation of A_i. We know already that $\beta_i A_i + \gamma_i \mathrm{id}_X$ represents a market invariant under price-scaling for any $\beta_i, \gamma_i \in \mathbb{C}$. Since A_i is skew-adjoint and $\beta_i A_i + \gamma_i \mathrm{id}_X$ needs to be an observable, we get that $\beta_i = i\mu_i$ and $\gamma_i = \omega_i$ for some $\mu_i, \omega_i \in \mathbb{R}$. Furthermore, since scaling of one price does not influence scaling of the others (i.e., $[p_i, U_j(\alpha)] = 0$ for $i \neq j$) we can use (2) and obtain

$$[p_i, i\mu_i A_j - \omega_i \mathrm{id}_X] = i\mu_i p_i \delta_{i,j}.$$

The operator $i\mu_i A_i + \omega_i \mathrm{id}_X$ is an observable, satisfies the same commutator relations as d_i respectively z_i and is invariant under price-scaling. Economic intuition therefore leads us to identify this operator with the demand respectively excess demand for good i if $\mu_i \neq 0$. The real parameter ω_i is identified as endowment. The other real parameter μ_i, the demand-supply gap, represents a new feature. Intuitively it measures the difference of first selling and then buying a good versus first buying and then selling that good. In the case $\mu_i = 0$, i.e., in classical general equilibrium theory, this existence result for market invariants generally has to be taken as an assumption [see, e.g. Mas-Colell, Whinston and Green (1995, p. 23)].

So far we have seen that in the case $\mu \neq 0$ at least one of the observables

p_i and d_i cannot be bounded. This fact vaguely resembles quantum theory, where position and momentum operators cannot simultaneously be bounded [see Wielandt (1949)]. Albeit, one must emphasize, that the reason for this unboundedness lies in the corresponding commutator relations and these relations are quite different, $[A, B] = A$ in our axiomatic framework and $[A, B] = \mathrm{id}_X$ in quantum theory. From a purely axiomatic viewpoint one might ask why this is the case. Since in economics we need demand invariance under price-scaling the right-hand side of **(MA4)** has to be, at least formally, positively homogeneous of degree one in p_i. Furthermore, by considering the adjoint of (1) a reasonable right-hand side has to be, at least formally, a skew-adjoint operator. Under the assumptions that the right-hand side is, e.g. a formal power series in p_i, the form $i\mu_i p_i$ we have chosen in **(MA4)** is unique. Hence, the system of axioms **(MA1)** - **(MA4)** is the only one satisfying the economic intuition of demand invariance under price-scaling. In quantum physics the intuition is that momentum is invariant under translation of position. To achieve this the right-hand side in the respective commutator relation can be chosen independent of position and momentum. Under the assumption that the right-hand side is, e.g. a formal power series one can show that $i\mu \mathrm{id}_X$ is unique to satisfy intuition and fit experimental evidence. If one now compares both right-hand sides, i.e., $i\mu_i p_i$ and $i\mu \mathrm{id}_X$, one observes that we do not propose the existence of a demand-supply gap independent of the goods under consideration. To the knowledge of the authors there is so far no experimental evidence for such a claim. That fact and the dependence on the individual price make things, at least formally, more complicated in axioms **(MA1)** - **(MA4)** compared to the corresponding axioms from quantum theory.

We close this section by providing a representation for the observables p_i, d_i and z_i on an appropriate Hilbert space. The results in the previous section, lead to the following approach: The Hilbert space is given as $X = L^2(\mathbb{R}^n)$. For a vector $x := (x_1, \ldots, x_n) \in \mathbb{R}^n$ and a function $\xi \in X$ the demand $d_i : D(d_i) \to X$ is given as a differential operator

$$d_i \xi = -i\mu_i \frac{d}{dx_i} \xi$$

with domain

$$D(d_i) = \{\xi \in X : \xi \text{ absolutely continuous and } \xi' \in X\}.$$

The excess demand operator $z_i = d_i - \omega_i \mathrm{id}_X$ has the same domain as d_i. Define the function $e_i : \mathbb{R}^n \to \mathbb{R}$ as $e_i(x) = e^{x_i}$. Then, the price operator $p_i : D(p_i) \to X$ is given as a multiplication operator

$$p_i \xi = e_i \cdot \xi$$

with domain

$$D(p_i) = \{\xi \in X : e_i \cdot \xi \in X\}.$$

All operators p_i, d_i, z_i are self-adjoint, p_i is positive, the commutator satisfies

$$[p_i, z_i]\,\xi = [p_i, d_i]\,\xi = -i\mu_i \left(e_i \cdot \frac{d}{dx_i}\xi - e_i \cdot \xi - e_i \cdot \frac{d}{dx_i}\xi \right) = i\mu_i p_i \xi$$

and thus the market axioms are fullfilled.

5 Observability

Since we assumed in **(MA4)** that p_i and d_i do not necessarily commute there is no simultaneous sharp measurement. Thus wealth (i.e., classically $\sum_{i=1}^n p_i d_i$) cannot be observed in a precise sense, and actually cannot even be defined in a naive way. That is a consequence of the fact that in asymmetric markets you cannot simultaneously keep the good and determine its price. You have to sell the good to determine its price and thus you change your wealth level. This interaction cannot be circumvented. Moreover, we derive the following

Proposition 3. For a market in state ξ the dispersions of p_i and d_i satisfy

$$\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} \geq \frac{\mu_i^2}{4} \left\| \sqrt{p_i}\xi \right\|^4 .$$

In the asymmetric case $\mu_i \neq 0$, the right-hand side is strictly larger than zero.

Proof. Since dispersion and mean do not depend on the norm of a state we can, without loss of generality, assume that $\|\xi\| = 1$ and obtain

$$\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} = \left\langle (p_i - \overline{p_{i\xi}}\mathrm{id}_X)^2 \xi|\xi \right\rangle \left\langle \left(d_i - \overline{d_{i\xi}}\mathrm{id}_X\right)^2 \xi|\xi \right\rangle .$$

Now Cauchy Schwarz inequality implies

$$\begin{aligned}\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} &\geq \left\langle (p_i - \overline{p_{i\xi}}\mathrm{id}_X)\left(d_i - \overline{d_{i\xi}}\mathrm{id}_X\right) \xi|\xi \right\rangle \\ &\quad \times \left\langle \left(d_i - \overline{d_{i\xi}}\mathrm{id}_X\right)(p_i - \overline{p_{i\xi}}\mathrm{id}_X) \xi|\xi \right\rangle .\end{aligned}$$

Since $ab = \frac{1}{2}[a,b]_+ + \frac{1}{2i} i\,[a,b]$ with $[a,b]_+ = ab + ba$ we obtain

$$\begin{aligned}\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} &\geq \left\langle \frac{1}{2}\left[d_i - \overline{d_{i\xi}}\mathrm{id}_X, p_i - \overline{p_{i\xi}}\mathrm{id}_X\right]_+ \xi|\xi \right\rangle^2 \\ &\quad + \left\langle \frac{1}{2i}\left[d_i - \overline{d_{i\xi}}\mathrm{id}_X, p_i - \overline{p_{i\xi}}\mathrm{id}_X\right] \xi|\xi \right\rangle^2\end{aligned}$$

and since the first term is positive

$$\begin{aligned}\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} &\geq \left\langle \frac{1}{2i}\left[d_i - \overline{d_{i\xi}}\mathrm{id}_X, p_i - \overline{p_{i\xi}}\mathrm{id}_X\right] \xi|\xi \right\rangle^2 \\ &\geq \left\langle \frac{1}{2i}[d_i, p_i]\, \xi|\xi \right\rangle^2 .\end{aligned}$$

Now (1) and the fact that positive oberservables have a square root yields the final inequality

$$\overline{(\triangle p_i)^2_\xi}\,\overline{(\triangle d_i)^2_\xi} \geq \frac{\mu_i^2}{4}\left\|\sqrt{p_i}\xi\right\|^4 .$$

Since $\mu_i \left\|\sqrt{p_i}\xi\right\|^4$ can only be zero if μ_i is zero the proposition is proved. ♢

As a consequence of this proposition we obtain that wealth cannot be measured sharply and hence is not observable. Therefore, there are difficulties to define classical budget sets restricting the consumption alternatives of agents to a given wealth level. The following observable

$$W(p, d) = \frac{1}{2} \sum_{i=1}^{n} [p_i, d_i]_+.$$

represents wealth in asymmetric markets and reduces to the usual definition for $\mu_i = 0$. Then the budget set for a price p and a wealth level $0 \le w \in \mathbb{R}$ can be defined as the following set of demands

$$B_{p,w,\xi} = \left\{ d : \overline{W(p, d)_\xi} \le w \right\}$$

Classically one now introduces preference relations or utility functions u and solves the maximization problem.

Problem 4. (Utility Maximization Problem) Find $d \in B_{p,w,\xi}$ such that $\overline{u(d)_\xi}$ is maximal. A solution $d(p, w, \xi)$ of this problem is called a Walrasian demand function.

In the symmetric case a straightforward compactness argument yields existence of Walrasian demand functions under suitable general assumtions on the utility function. In the asymmetric case it is sufficient for our purposes to assume existence for the remainder of this section. Now given such a Walrasian demand function $d(p, w, \xi)$ the Walrasian excess demand function is defined as

$$z_i(p, \omega, \xi) = d_i(p, \overline{W(p, \omega)_\xi}, \xi) - \omega_i \mathrm{id}_X,$$

where $\overline{W(p, \omega)_\xi}$ measures wealth provided through endowment. Now, under suitable assumptions on the utility function the Walrasian equilibrium price vector p^w clears all markets, i.e.,

$$z_i(p^w, \omega, \xi)\xi = 0 = d_i(p^w, \overline{W(p^w, \omega)_\xi}, \xi) - \omega_i \mathrm{id}_X$$

for all goods i with $0 \leq i \leq n$. Thus, in a Walrasian equilibrium state ξ the demand d_i of a good is fixed to be the endowment ω_i and hence its dispersion satisfies $\overline{(\triangle d_i)^2_\xi} = 0$. Proposition 3 now implies $\mu_i = 0$ and the main result of this section.

Theorem 5. In an asymmetric exchange market Walrasian equilibrium prices do not exist.

As one could see in the derivation of this result, even in the symmetric case, there are several necessary assumptions to obtain the existence of an equilibrium. Key to the argumentation usually is that the relation between price and demand is first a function and has second further properties like continuity or negative derivative. While there seems to be sufficent experimental evidence for the function property, the other properties are often relaxed [see, e.g., Hart (1975) for a singular non-existence and Momi (2001) for a generic non-existence result] and no equilibrium is found. Markets without equilibrium are thus a vivid field of research. Theorem 5 now fits into this framework as follows: Relation (1) between price and demand is only just sufficient to model an endowment effect and to obtain reasonable market invariants under price-scaling. However, it implies Theorem 5 and thus excludes the existence of an equilibrium for any choice of utility. In other words, as long as there is an endowment effect in the market, one can choose an arbitrarily "tame" relation between price and demand and will still not get an equilibrium. Combining this with the results of the prior sections we obtain our final assertion. The non-existence of an equilibrium price in an asymmetric market does not necessarily come from a bad choice of utility or a bad choice of preference, it originates from the endowment effect.

6 Conclusion

Our axiomatic framework for exchange markets is complementary to classical approaches. If the endowment effect is sufficiently small, i.e. if the modulus of μ_i is sufficiently small in equation (1), the asymmetric market is very likely in a market state which can under suitable assumptions be approximated by the Walrasian equilibrium of the corresponding symmetric market. However, if we consider an asymmetric market with a large demand-supply gap, i.e. , the endowment effect is prevalent, our results differ from what one would expect from classical general equilibrium theory. In contrast to general equilibrium theory we obtain: First, demand invariance under price-scaling has not to be assumed. We show that the proposed axioms for an exchange economy are unique to support an economically reasonable identification of the market invariants with demand and excess demand. Second, our representations of price and demand are unbounded operators on an infinite dimensional Hilbert space. We prove that neither can this space be finite dimensional nor can these operators be bounded. Third, there is no Walrasian equilibrium.

Our results indicate that classical general equilibrium theory may actually be seen as a valid framework in the context of symmetric or asymmetric markets with a sufficiently small demand-supply gap. However, this is not the case for asymmetric markets with a sufficiently large demand-supply gap.

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