Wolfgang Leininger

Evolutionarily Stable Preferences in Contests

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Abstract
We define an indirect evolutionary approach formally and apply it to (Tullock)contests. While it is known (Leininger, 2003) that the direct evolutionary approach in the form of finite population ESS (Schaffer, 1988) yields more aggressive behavior than in Nash equilibrium, it is now shown that the indirect evolutionary approach yields the same more aggressive behavior, too. This holds for any population size *N*, if evolution of preferences is determined by behavior in two-player contests. The evolutionarily stable preferences (ESP) of the indirect approach turn out to be negatively interdependent, thereby "rationalizing" the more aggressive behavior.

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1 Introduction

Contest models based on a family of contest success functions, which was introduced by Tullock (1980), have become a standard workhorse of contest and conflict theory; this is true for both, the growing number of economic and non-economic areas of applications of contest theory as well as the theoretical advancement of contest theory itself (see e.g. the survey volumes edited by Lockard and Tullock (2001) and - most recently - Congleton, Hillman and Konrad (2008)). The present paper hence addresses a new theoretical question on purpose in the framework of Tullock's model without striving for utmost generality of results.

Most of the theoretical body of contest theory, and in particular rent-seeking theory, is based on the solution concept Nash equilibrium. Only recently have evolutionary solution concepts like the notion of an evolutionarily stable strategy (ESS) successfully entered the field, in particular in the notion of Schaffer's (1988) adaptation of the standard notion of an ESS (Maynard-Smith, 1974) to finite population of players. While the latter amounts to a refinement of Nash equilibrium, the former can, but need not, be different from Nash equilibrium. Leininger (2003) shows, that for a large class of contest success functions, including Tullock contest success functions, the two solution concepts must always give different solutions. Hehenkamp, Leininger and Possajennikov (2004) provide a full ESS-analysis of Tullock's original contest model and relate the results to those of the Nash-based theory. Among the most striking results is the observation, that overdissipation of the rent can be an ESS-outcome of the contest. Further analyses of contests, which use Schaffer's concept are contained in Leininger (2006), Schmidt(2007), and Hehenkamp (2007). All these papers refer to evolution of *strategies*, respectively behavior; i.e. the solution concept ESS is applied to the strategy sets of a game with fixed preferences, which could be termed the standard (or *direct*) evolutionary approach. In contrast, Güth and Yaari (1992) have pioneered (and termed) an *indirect* evolutionary approach, in which evolution operates at the level of *preferences*, while actions are still determined by Nash equilibrium w.r.t. fixed strategy sets. The quest then is for evolutionarily stable preferences (ESP). Güth and Peleg (2001) have addressed this question w.r.t. Maynard-Smiths' notion of ESS by searching for conditions under which standard payoff (maximization) is evolutionarily stable, while Wärneryd (2008) has identified evolutionarily stable preferences w.r.t. Schaffer's ESS notion in a contest model, in which cost parameters evolve evolutionarily.

The present paper goes one step further by investigating the relation between behavior determined by the direct evolutionary approach at the level of strategies and the indirect evolutionary approach at the level of preferences. We develop both approaches in detail and apply them to simple versions of an indirect evolutionary

Tullock contest. As is standard in evolutionary game theory we mainly concentrate on two-player evolutionary games, which, however, are played by populations of arbitrary size. For each population size we determine evolutionarily stable preferences (ESP) uniquely and relate Nash equilibrium behavior w.r.t. ESP to behavior which results from applying ESS to strategies directly according to the direct evolutionary approach. Our main finding is that both approaches determine the *same* behavior for all sizes of the population.

The paper is organized as follows: Section 2 summarizes Nash solution theory (Tullock 1980) and ESS solution theory (Hehenkamp et al. 2004) of Tullock contests. Section 3 introduces the indirect evolutionary approach, while section 4 gives an illustrative example of a Tullock contest, in which non-standard preferences yield an evolutionary advantage over standard preferences. Section 5 presents the indirect evolutionary game and its solution concept in detail. Section 6 contains the main body of analysis of two-player evolutionary games with variable population size. Section 7 concludes.

2 Nash behavior and evolutionarily stable behavior

Recall that Tullock (1980) proposed a model, in which n rent-seekers compete for a rent of size V. If the contestants expend $x = (x_1, ..., x_n)$, $x_i \geq 0$, the probability of success for player i, $i = 1, ..., n$ is given by

$$p_i(x_1, ..., x_n) = \frac{x_i^r}{\sum_{j=1}^{n} x_j^r}$$

and expected profit for player i is given by

$$\Pi_i(x_1, ..., x_n) = p_i(x_1, ..., x_n)V - x_i = \frac{x_i^r}{\sum_{j=1}^{n} x_j^r}V - x_i.$$

One can show that for $r \leq n/(n-1)$, a unique Nash equilibrium in pure strategies exists in this game, in which each player maximizes expected payoff by bidding

$$x^* = \frac{n-1}{n^2} rV.$$

Aggregate rent-dissipation then amounts to

$$nx^* = \frac{n-1}{n} rV.$$

Equilibrium expenditures never exceed V, the value of the rent, but may be strictly less than V. The "full rent dissipation"-hypothesis does not hold; yet overdissipation is incompatible with individually rational payoff maximization as it would imply, that at least one contestant has a negative payoff in equilibrium (and would therefore be better off by non-participation or bidding zero).

Later Hehenkamp, Leininger and Possajennikov (2004) used the concept of an evolutionarily stable strategy (ESS) for finite population (Schaffer, 1988) to reanalyse this game.

A strategy is evolutionarily stable, if a whole population using that strategy cannot be invaded by a sufficiently small group of "mutants" using another strategy. Similarly, a standard of behavior in an economic contest is evolutionarily stable, if - upon being adopted by all participants in the contest - no small subgroup of individuals using a different standard of behavior can invade and "take over". Obviously, in the context of finite populations the smallest meaningful number of mutants is one. The emphasis of the evolutionary approach is not on explaining action (as a result of particular choice or otherwise), but on the diffusion of forms of behavior in groups (as a result of learning, imitation, reproduction or otherwise).

The definition of invadability is all important:

Definition 1:

i) Let a strategy (standard of behavior) x be adapted by all players i, $i = 1, ..., n$. A mutant strategy $\bar{x} \neq x$ can invade x, if the payoff for a single player using $\bar{x}$ (against x of the $(n-1)$ other players) is strictly higher than the payoff of a player using x (against $(n-2)$ other players using x and the mutant using $\bar{x}$).

ii) A strategy x^{ESS} is evolutionarily stable, if it cannot be invaded by any other strategy.

Roughly speaking, an ESS is such that, if almost all members of a group adopt it, there is no other strategy that could give a higher relative payoff, if used by a group member. The dynamic justification for this notion of equilibrium is, that more successful strategies diffuse or "reproduce" faster than less successful ones and ultimately extinguish the latter. Formally, consider a group of N potential contestants, who may recurrently engage in a contest of size $n < N$; i.e. only n out of N contestants compete in the contest. The expected payoff of a single mutant among the N contestants if he is drawn into a contest and plays a strategy $\bar{x}$ against $(n-1)$ other players using strategy x is then still given by

$$\Pi_1(\bar{x}, x, ..., x) = \frac{\bar{x}^r}{(n-1)x^r + \bar{x}^r} V - \bar{x};$$

but one of the other $(n-1)$ players i, $i \in \{2, ..., n\}$, chosen for the contest expects

$$\Pi_i = (1 - \tfrac{n-1}{N-1})\Pi_i(x, ..., x) + \tfrac{n-1}{N-1}\Pi_i(\bar{x}, x, ..., x)$$

as the probability, that a chosen player i will face the mutant player 1 from the remaining $(N-1)$ potential players among the further chosen $(n-1)$ players is $(n-1)/(N-1)$. Note that we have assumed that players are chosen for participation randomly and with equal probability. Consequently, an ESS strategy x^{ESS} must now solve the problem (Schaffer, 1988) of relative payoff maximization

$$\max_x \Pi_1(x, x^{ESS}, ..., x^{ESS}) - (1 - \tfrac{n-1}{N-1})\Pi_i(x^{ESS}, ..., x^{ESS})$$
$$- \tfrac{n-1}{N-1}\Pi_i(x, x^{ESS}, ..., x^{ESS})$$

Eliminating the constant term $(1-(n-1)/(N-1))\Pi_i(x^{ESS}, ..., x^{ESS})$ equivalently yields

$$\text{(E)} \qquad \max_x \Pi_1(x, x^{ESS}, ..., x^{ESS}) - \tfrac{n-1}{N-1}\Pi_i(x, x^{ESS}, ..., x^{ESS}).$$

We can directly read off from the maximand, that as $N \to \infty$ we approach the Nash equilibrium problem and hence the difference in behavior among n contestants in Nash equilibrium and among n contestants (chosen out of large population of potential contestants) in evolutionary equilibrium disappears.

Hehenkamp et al. (2004, Theorem 5) show that for $r \leq n/(n-1)$ a unique ESS exists, in which each player maximizes relative payoff by bidding

$$\text{(DE)} \qquad x^{ESS} = \tfrac{n-1}{n^2}\tfrac{N}{N-1}rV.$$

Hence a unique ESS exists under precisely those circumstances, which imply existence of a Nash equilibrium (NE) in pure strategies. Both concepts predict different behavior for a population of $N = n$ players! ESS behavior is more aggressive as $x^{ESS} = \frac{r}{n}V$ always exceeds $x^* = \frac{(n-1)r}{n^2}V$. Moreover, aggregate ESS-behavior does not depend on the number of players involved in the contest, only on the value of the price V and the contest technology parameter r as $n \cdot x^{ESS} = r \cdot V$. This means over-/under-dissipation of the rent if $r \gtrless 1$; in case $r = 1$ full rent dissipation applies.

3 The indirect evolutionary approach

The indirect evolutionary approach (Güth and Yaari, 1992) assumes that behavior evolves evolutionarily indirectly; i.e. the evolutionary forces do not work on actions or strategies directly, but on entities or "stimuli" which in turn determine or at least influence behavior. A quite general and economically highly relevant case occurs when such genetically determined stimuli shape individual *preferences* of players. Evolutionary forces then work towards preference evolution rather than behavior (or strategy) evolution directly.

The indirect evolutionary approach assumes a large population of players, who play an evolutionary game as follows: For expositional clarity let us focus on two-player contests. From the population pairs of players are randomly drawn into playing the contest, in which they non-cooperatively interact by choosing a strategy each, which determines outcome and payoffs of the game. Moreover, the outcome of the game also determines evolutionary fitness according to a "fitness function", which need not be identical to the payoff function of the players. This is the key insight of the indirect approach that success in short terms of payoff need not coincide with evolutionary success in the long-run. If, however, players differ with respect to evolutionary success, the more successful will spread faster within the population than the less successful and hence their "stimuli" resp. preferences will become more frequent in the population. This triggers an evolutionary process in the space of preferences, which determines in the long-run a distribution of evolutionarily stable preferences, which then in turn determine a stable distribution of behavior (as the process of preference evolution has come to a halt). It is this behavior, which results from indirect evolution, which we want to compare and relate to behavior resulting from direct evolution of strategies (as determined by ESS according to the Definition 1).

More precisely, we shall assume that any two players with whatever preferences (to be specified later) play according to Nash equilibrium when drawn into a contest. I.e. they behave *rationally*, which may be the result of a learning process. The Nash equilibrium with respect to the given preferences then determines the evolutionary success of both players (resp. preferences) according to the fitness function. This fitness function is identified with economic, i.e. material success or payoff which results from the differences between prize(s) won and effort or bids expended. The indirect evolution of preferences then resembles incentive structures from strategic commitment or delegation problems, in that direct (material) payoff maximization may lead to lower evolutionary success than the maximization of other preferences, which might emerge in the evolutionary process. One strand of the literature poses independent preferences (i.e. material pay off) against interdependent preferences (i.e. those which also express positive or negative concern for others) and ask for strategic advantages of one over the other (see Kockesen et al. (2000 a, b) and for contests in particular Guse and Hehenkamp (2006)). This literature, however, does not ask, let alone answer the question of evolutionary stability of preferences; it states advantages in material terms of one preference type over another. The next section illustrates this point.

4 A simple (motivational) example

Consider a Tullock contest with just two players and a conflict technology parameter $r = 1$, in which player 1 is an *absolute* payoff maximizer, while player 2 maximizes *relative* payoff (a particular form of interdependent preferences):

$$\begin{aligned} \Pi_1(x_1, x_2) &= \tfrac{x_1}{x_1+x_2}V - x_1 \qquad \text{and} \\ \Pi_2(x_1, x_2) &= (\tfrac{x_2}{x_1+x_2}V - x_2) - (\tfrac{x_1}{x_1+x_2}V - x_1) \\ &= \tfrac{x_2-x_1}{x_1+x_2}V - (x_2 - x_1) \end{aligned}$$

Then it is easily calculated that the unique Nash equilibrium is given by

$$x_1^* = \tfrac{2}{9}V \qquad \text{and} \qquad x_2^* = \tfrac{4}{9}V$$

yielding payoffs of $\Pi_1^* = \frac{1}{9}V$ and $\Pi_2^* = \frac{1}{9}V$.

So, due to his relative concern player 2 spends twice as much as player 1.

Suppose now that for evolutionary success the "material" payoff of each player is decisive; i.e. the evolutionary success or "fitness" function is given by the absolute payoff $\overline{\Pi}_i(x_1, x_2) = \frac{x_i}{x_i+x_j}V - x_i$ with $i, j = 1, 2$, $i \neq j$.

Then the Nash equilibrium behavior (x_1^*, x_2^*) determines material payoffs as

$$\overline{\Pi}_1(x_1^*, x_2^*) = \tfrac{1}{9}V$$

$$\overline{\Pi}_2(x_1^*, x_2^*) = \tfrac{2}{9}V.$$

Consequently, the player with interdependent preferences is evolutionarily more successful than the player with independent strategies! Does this mean that independent preferences - i.e. classical *homo oeconomicus* - cannot be evolutionarily stable? As we shall show, this depends on the size of the population, which plays the indirect evolutionary game. For this we have to specify this game in detail.

5 The indirect evolutionary game

We now specify an evolutionary game in full detail by following ideas of Güth and Peleg (2001). As is standard in evolutionary game theory (see e.g. Weibull, 1995) we look at symmetric 2-player games.

Denote by $M = M_1 \times M_2$ the "mutation space" of stimuli, which determines preferences parametrically. In the literature on delegation these would be called "types" of players; and let $S = S_1 \times S_2$ denote the strategy sets of players 1 and 2.

An individual preference Π_i is then a function

$$\Pi_i : S \times M \to \mathbb{R} \qquad i = 1, 2,$$

i.e. payoffs are determined by types and actions.

However, types and actions result via separate processes. Types evolve through an evolutionary process while action choice is always the result of (Nash-) equilibrium behavior in a game with given types.

Let $f_i : S \to \mathbb{R}$, $i \in N$ denote the evolutionary success or 'fitness' function for player i where N denotes the player population of the evolutionary game. Note, that fitness of a player is the result of behavior only and does not - at least not directly - depend on the types of players.

Whenever two players from N with types m_1 and m_2 are randomly drawn into a contest, they play the following game $\bar{G}$:

$$\bar{G}(m_1, m_2) = (\{1, 2\}, S, \Pi_1(\cdot, \cdot, m_1, m_2), \Pi_2(\cdot, \cdot, m_1, m_2))$$

$\bar{G}$ is well-defined for any player set $\{1, 2\} \subset N$.

Any equilibrium $((x_1^*(m_1, m_2), x_2^*(m_1, m_2))$ of $\bar{G}(m_1, m_2)$ - in our case equilibrium is always unique - determines not only equlibrium payoffs $(\Pi_1^*(x_1^*(m_1, m_2), x_2^*(m_1, m_2), m_1, m_2), \Pi_2^*(x_1^*(m_1, m_2), x_2^*(m_1, m_2), m_1, m_2))$ but also evolutionary fitness - or *material* payoff - of the players as $(f_1(x_1^*(m_1, m_2), x_2^*(m_1, m_2)), f_2(x_1^*(m_1, m_2), x_2^*(m_1, m_2)))$.

This gives rise to *indirect* fitness functions

$$F_i(m_1, m_2) = f_i(x_1^*(m_1, m_2), x_2^*(m_1, m_2)) \qquad i = 1, 2$$

for the players, which can be regarded as the payoff functions of an evolutionary game, which is played over types resp. preferences. Denote this game by

$$G = (\{1,2\}, M, F_1(m_1,m_2), F_2(m_1,m_2))$$

and the solution concept applied to G is ESS as defined above. Since an evolutionarily stable strategy of this game is a type; i.e. a *preference*, we call an ESS of G in the following ESP, which stands for evolutionarily stable preference.

Hence a solution of the (full) indirect evolutionary game is given by a vector of types and strategies

$$(m^*, x^*(m^*)) = ((m_1^*, m_2^*), (x_1^*(m_1^*, m_2^*), x_2^*(m_1^*, m_2^*))$$

such that

i) $m^* = (m_1^*, m_2^*)$ is an ESP of G and

ii) $x^*(m^*) = (x_1^*(m_1^*, m_2^*), x_2^*(m_1^*, m_2^*))$ is a Nash equilibrium of $\bar{G}(m_1^*, m_2^*)$.

This configuration is a stationary one, if we think of evolution as a process (which we have not modelled), that determines the share of induviduals of type m_i in the overall population at stage $(t+1)$ as a function of types and their realized fitness in period t. In particular, $x^*(m^*)$ is the (indirectly) evolutionarily stable behavior.

6 Evolutionarily stable Preferences in 2-player Contests

We now specify a two-player Tullock contest, which is played by a population of contestants whose preferences can be of the following interdependent type:

$$\Pi_1(x_1, x_2|\alpha_1) = \tfrac{x_1}{x_1+x_2}V - x_1 - \alpha_1(\tfrac{x_2}{x_1+x_2}V - x_2) \qquad \text{and}$$

$$\Pi_2(x_1, x_2|\alpha_2) = \tfrac{x_2}{x_1+x_2}V - x_2 - \alpha_2(\tfrac{x_1}{x_1+x_2}V - x_1)$$

with $\alpha_1 \in [-1,1]$ and $\alpha_2 \in [-1,1]$. Hence $M = M_1 \times M_2 = [-1,1] \times [-1,1]$ and $S = S_1 \times S_2 = \mathbb{R}_+ \times \mathbb{R}_+$.

The stimuli α_i, $i = 1,2$, can be interpreted as determining "social attitude" of a single player: preferences with $\alpha_i < 0$ can be termed altruistic, those with $\alpha_i > 0$ are spiteful, while $\alpha_i = 0$ refers to the independent type of pure self-interest. Accordingly preferences with $\alpha_i < (>)\ 0$ are also termed positively (negatively) interdependent.

So let us analyse the game $G(\alpha_1, \alpha_2)$, in which the two contestants have preferences

$$\pi_1(x_1, x_2) = \frac{x_1 - \alpha_1 x_2}{x_1 + x_2} V - (x_1 - \alpha_1 x_2)$$

$$\pi_2(x_1, x_2) = \frac{x_2 - \alpha_2 x_1}{x_1 + x_2} V - (x_2 - \alpha_2 x_1)$$

It is then immediate that

Lemma 1:

The unique Nash equilibrium of $G(\alpha_1, \alpha_2)$ is given by

$$x_1^*(\alpha_1, \alpha_2) = \frac{1+\alpha_2}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2} V \quad \text{and} \quad x_2^*(\alpha_1, \alpha_2) = \frac{1+\alpha_1}{(1+\frac{1+\alpha_1}{1+\alpha_2})^2} V.$$

Note that $x_1^* = \frac{1+\alpha_1}{1+\alpha_2} \cdot x_2^*$ and hence $x_1^* + x_2^* = \frac{1+\alpha_1+\alpha_2+\alpha_1 \cdot \alpha_2}{2+\alpha_1+\alpha_2} V < V$ as $\alpha_1 \cdot \alpha_2 < 1$, if $\alpha_i \in (-1, 1)$. So whatever the two types of contestants - altruistic preferences meeting spiteful ones or otherwise - there is always incomplete dissipation of the rent at stake. Equilibrium payoffs to both players are always equal:

$$\Pi_1^* = \Pi_2^* = \frac{(1-\alpha_1\alpha_2)^2}{(2+\alpha_1+\alpha_2)^2} V;$$

i.e. the equality of payoffs in our motivating example $\bar{G}(0, 1)$ is a general feature over *all* games $\bar{G}(m_1, m_2)$, $(m_1, m_2) \in M$.

Also observe that a more spiteful player behaves more aggressively against any kind of opponent as $\frac{\partial x_i^*}{\partial \alpha_i} > 0$ for all $\alpha_j \in (-1, 1)$, $i, j = 1, 2$, $i \neq j$. Does this give spiteful players the edge on less spiteful players in the game of preference evolution?

The answer to this question is given by an analysis of the resulting material payoffs from equilibrium play $x_1^*(\alpha_1, \alpha_2)$ and $x_2^*(\alpha_1, \alpha_2)$. Material payoffs are given by

$$f_1(x_1, x_2) = \frac{x_1}{x_1 + x_2} V - x_1 \qquad \text{and}$$

$$f_2(x_1, x_2) = \frac{x_2}{x_1 + x_2} V - x_2$$

and hence

$$F_1(\alpha_1, \alpha_2) = \frac{x_1^*(\alpha_1,\alpha_2)}{x_1^*(\alpha_1,\alpha_2) + x_2^*(\alpha_1,\alpha_2)} V - x_1^*(\alpha_1, \alpha_2)$$

$$F_2(\alpha_1, \alpha_2) = \frac{x_2^*(\alpha_1,\alpha_2)}{x_1^*(\alpha_1,\alpha_2) + x_2^*(\alpha_1,\alpha_2)} V - x_2^*(\alpha_1, \alpha_2).$$

Inserting the expressions for $x_1^*(\alpha_1, \alpha_2)$ and $x_2^*(\alpha_1, \alpha_2)$ according to Lemma 1 yields

$$F_1(\alpha_1, \alpha_2) = \frac{\frac{1-\alpha_1 \cdot \alpha_2}{1+\alpha_1}}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2} V = \frac{\frac{1+\alpha_2}{1+\alpha_1} - \alpha_2}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2} V$$

$$F_2(\alpha_1, \alpha_2) = \frac{\frac{1-\alpha_1 \cdot \alpha_2}{1+\alpha_2}}{(1+\frac{1+\alpha_1}{1+\alpha_2})^2} V == \frac{\frac{1+\alpha_1}{1+\alpha_2} - \alpha_1}{(1+\frac{1+\alpha_1}{1+\alpha_2})^2} V.$$

Note again, that $F_1(\alpha_1, \alpha_2) = \frac{1+\alpha_1}{1+\alpha_2} F_2(\alpha_1, \alpha_2)$; i.e. the fitness of player 1 is higher if and only if $\alpha_1 > \alpha_2$.

More importantly, $F_1(\alpha_1, \alpha_2)$ and $F_2(\alpha_1, \alpha_2)$ are symmetric in the sense that $F_1(\alpha_1, \alpha_2) = F_2(\alpha_2, \alpha_1)$ for all $(\alpha_1, \alpha_2) \in [-1, 1]^2$. Note that at this stage we can safely exclude "perfect" altruists; i.e. $\alpha_1 = -1$ resp. $\alpha_2 = -1$, from further consideration. A perfect altruist would bid $x^* = 0$ against *any* opponent. Any opponent would reply *in equilibrium* with a bid of 0. But this means that an opponent with $\alpha > -1$ ends up with positive fitness of V, whereas the perfect altruist obtains fitness 0 (e.g. $\lim_{\alpha_2 \to -1} F_1(\alpha_1, \alpha_2) = V$ and $\lim_{\alpha_2 \to -1} F_2(\alpha_1, \alpha_2) = 0$, if $\alpha_1 > -1$). The case of two perfect altruists is indeterminate with regard to resulting fitness and can hence not satisfy the stability criterion of evolutionary equilibrium.

We now solve the symmetric game $G = (M, F) = ([-1, 1] \times [-1, 1], F_1(\alpha_1, \alpha_2), F_2(\alpha_1, \alpha_2))$. In order to do this we have to specify the population of players playing the indirect evolutionary game, two cases are considered:

i) Infinite Population

Suppose the population of contestants is infinitely large. We then define the concept of evolutionarily stable preferences (ESP) in terms of the preference parameters α as a symmetric Nash equilibrium, which is stable against invasion of another preference.

Definition 2:

$\alpha^* \in [-1, 1]$ yields evolutionarily stable preferences, if (α^*, α^*) satifies

i) $F_1(\alpha^*, \alpha^*) \geq F_1(\alpha_1, \alpha^*)$ for all $\alpha_1 \in [-1, 1]$ and

ii) $F_1(\alpha^*, \alpha^*) > F_1(\alpha, \alpha)$ for all $\alpha \in [-1, 1]$ which satisfy $F_1(\alpha^*, \alpha^*) = F_1(\alpha, \alpha^*)$.

Note that because of symmetry this implies the same inequalities with respect to F_2. Hence (α^*, α^*) must be a symmetric Nash equilibrium, which satisfies the stability condition ii). We now prove

Proposition 1:

For an infinite population the unique symmetric Nash equilibrium of G is given by $(\alpha^*, \alpha^*) = (0, 0)$; i.e. the evoutionarily stable preferences are given by

$\Pi_1(x_1, x_2|0) = \frac{x_1}{x_1+x_2}V - x_1$ and $\Pi_2(x_1, x_2|0) = \frac{x_2}{x_1+x_2}V - x_2$

(Proof: see Appendix 1)

Hence material payoff or independent preferences turn out to be the unique ESP! In an infinite population *material* payoff maximization also leads to evolutionary success, a point already made by Alchian (1950). If we now compare behavior determined by the direct evolutionary approach and indirect evolutionary approach w.r.t. the same material payoff, we realize:

Theorem 1:

For an infinite population of players engaged in 2-player contests, ESS behavior with respect to material payoff (direct evolution) coincides with Nash behavior with respect to ESP (indirect evolution).

The Theorem expresses the fact that Nash equilibrium (if unique) and ESS coincide conceptually, if the population in the evolutionary game is infinite, *and* that material payoff is the evolutionarily stable preference, if the population in the evolutionary game is infinite.

Our Proposition 1 can then be related to Güth and Peleg (2001), who for such 2-player evolutionary games ask under what conditions payoff maximization (understood as *material* payoff maximization) will survive in the evolutionary game, if this is played by an infinite population. Our game satisfies their Theorem 7 (p. 487), which gives *necessary* conditions. More precisely, our game satisfies their technical assumption (II.10). However, their additional assumption (II.11), which would make (II.10) a sufficient condition is not satisfied everywhere in our type spaces.

ii) Finite Population

We now assume that the population of players in the evolutionary game, from which contestants are pairwise randomly drawn, is finite. Schaffer's (1988) definition of an ESS has then to be adapted to the preference level, again we call it for this reason an ESP.

Definition 3:

$\alpha^* \in [-1, 1]$ yields evolutionarily stable preferences (ESP), if (α^*, α^*) satisfies the condition that α^* maximizes

$F_1(\alpha, \alpha^*) - \frac{1}{N-1}F_2(\alpha, \alpha^*)$

Note that this condition simply results from (E), if $n = 2$.

So look at the following problem:

$\max_{\alpha_1} F_1(\alpha_1, \alpha_2) - \frac{1}{N-1} F_2(\alpha_1, \alpha_2)$ which reads

$$\max_{\alpha_1} \frac{\frac{1+\alpha_2}{1+\alpha_1} - \alpha_2}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2} V - \frac{1}{N-1} \cdot \frac{\frac{1+\alpha_1}{1+\alpha_2} - \alpha_1}{(1+\frac{1+\alpha_1}{1+\alpha_2})^2} V$$

The first-order condition of this problem is given by

$$\frac{(1+\frac{1+\alpha_2}{1+\alpha_1})^2(-\frac{1+\alpha_2}{(1+\alpha_1)^2}) - (\frac{1+\alpha_2}{1+\alpha_1} - \alpha_2)2\cdot(1+\frac{1+\alpha_2}{1+\alpha_1})(-\frac{1+\alpha_2}{(1+\alpha_1)^2})}{(1+\frac{1+\alpha_2}{1+\alpha_1})^4} V$$

$$- \frac{1}{N-1} \cdot \frac{(1+\frac{1+\alpha_1}{1+\alpha_2})^2(\frac{1}{1+\alpha_2} - 1) - (\frac{1+\alpha_1}{1+\alpha_2} - \alpha_1)(1+\frac{1+\alpha_1}{1+\alpha_2})\cdot 2 \cdot \frac{1}{1+\alpha_2}}{(1+\frac{1+\alpha_1}{1+\alpha_2})^4} V = 0.$$

Since we are looking for a symmetric solution we can set $\alpha_1 = \alpha_2 = \alpha$, which yields

$$\frac{4(-\frac{1}{1+\alpha}) - (1-\alpha)\cdot 4 \cdot (-\frac{1}{1+\alpha})}{16} - \frac{1}{N-1} \frac{4(\frac{1}{\alpha+1} - 1) - (1-\alpha)\cdot 4 \cdot \frac{1}{1+\alpha}}{16} = 0.$$

Hence it must hold that

$$-\frac{4}{1+\alpha} + 4 \cdot \frac{1-\alpha}{1+\alpha} - \frac{1}{N-1} \cdot \frac{4}{1+\alpha} + \frac{4}{N-1} + \frac{4}{N-1} \cdot \frac{1-\alpha}{1+\alpha} = 0$$

or - after multiplication by $\frac{(1+\alpha)}{4}$ -

$$-1 + (1-\alpha) - \frac{1}{N-1} + \frac{(1+\alpha)}{N-1} + \frac{1}{N-1}(1-\alpha) = 0$$
$$-\alpha - \frac{1}{N-1} + \frac{1}{N-1}(1+\alpha+1-\alpha) = 0$$
$$-\alpha - \frac{1}{N-1} + \frac{2}{N-1} = 0$$
$$-\alpha + \frac{1}{N-1} = 0$$
$$\Rightarrow \alpha = \frac{1}{N-1}.$$

We have proven the following

Proposition 2:

For a finite population of size N the unique evolutionary stable preferences (ESP) are given by $(\alpha^*, \alpha^*) = (\frac{1}{N-1}, \frac{1}{N-1})$; i.e.

$$\Pi_1(x_1, x_2, \frac{1}{N-1}, \frac{1}{N-1}) = \frac{x_1}{x_1+x_2} V - x_1 - \frac{1}{N-1}(\frac{x_2}{x_1+x_2} V - x_2)$$

$$\Pi_2(x_1, x_2, \frac{1}{N-1}, \frac{1}{N-1}) = \frac{x_2}{x_1+x_2} V - x_2 - \frac{1}{N-1}(\frac{x_1}{x_1+x_2} V - x_1).$$

Proposition 2 shows that the evolutionary stable preferences vary between *absolute* (or material) payoff; i.e. $(\alpha^*, \alpha^*) = (0, 0)$, in the case of an infinite population to *relative* payoff; i.e. $(\alpha^*, \alpha^*) = (1, 1)$, if the overall population is $N = 2$. Those are the maximal and minimal population sizes of the evolutionary game. In any case, ESP are negatively interdependent for all $2 \leq N < \infty$.

An immediate consequence of Proposition 2 is

Theorem 2:

> *For a finite population of size N of players engaged in 2-player contests ESS behavior with respect to material payoff (direct evolution) coincides with Nash behavior with respect to ESP (indirect evolution).*

Proof:

The direct evolutionary approach yields ESS behavior according to equation (DE) as $x^{ESS} = \frac{N}{4(N-1)}V$. The indirect evolutionary approach yields ESP as $(\frac{1}{N-1}, \frac{1}{N-1})$, which yields Nash equilibrium behavior according to Lemma 1 as

$$x^*(\tfrac{1}{N-1}, \tfrac{1}{N-1}) = \frac{1+\frac{1}{N-1}}{4}V = \frac{\frac{N}{N-1}}{4}V = \frac{N}{4(N-1)}V \qquad \text{q.e.d.}$$

Our Theorems can be summaried as

Corollary 1:

> *For evolutionary 2-player contest games direct evolution (of actions) and indirect evolution (of preferences) determine the same behavior of players, which only depends on the size of the population.*

7 Conclusion

Our investigation has produced an "equivalence" result: applying the direct evolutionary approach to Tullock's contest model is equivalent to applying the indirect evolutionary approach. Whatever the size of the population involved in two-player contests, the *same* behavior is determined by the evolutionary solutions ESS and ESP. We know from Leininger (2003) and Hehenkamp et al. (2004), that in case of a finite population the former yields more aggressive - spiteful - behavior than rational behavior according to Nash equilibrium w.r.t. independent preferences. The latter, which combines an evolutionary process at the preference level with rational choice at the action level, is now seen to determine negatively interdependent - spiteful - preferences, which "rationalize" (in Nash equilibrium) this increased aggression. With a fixed population size evolutionary forces at the preference level completely neutralize the change in the choice criterion at the action level (from ESS to NE), if preferences are endogenous to evolution.

Whether evolution works at the preference level or the action level it increases competition among contestants beyond the level that would apply without any evolution. With *endogenous* population sizes this would work towards a reduction in population sizes (because more 'fitness' is dissipated in contest interaction) and hence to the detriment of the population. This should - evolutionarily - favour groups who manage to suppress this tendency, a topic which we want to pursue in future research.

Appendix 1:

Proof of Proposition 1:

We have to search for an interior symmetric Nash equilibrium of the game with payoff functions $(F_1(\alpha_1, \alpha_2), F_2(\alpha_1, \alpha_2))$ and "strategies" $\alpha_1 \in [-1, 1]$ and $\alpha_2 \in [-1, 1]$.

Consider player 1 and his best response to any $\alpha_2 \in [-1, 1]$ of his opponent.

$\max_{\alpha_1 \in [-1,1]} F_1(\alpha_1, \alpha_2)$ yields the first-order condition

$$\frac{\partial}{\partial \alpha_1}\left(\frac{\frac{1-\alpha_1 \cdot \alpha_2}{1+\alpha_1}}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2}V\right) = \frac{\partial}{\partial \alpha_1}\left(\frac{\frac{1+\alpha_2}{1+\alpha_1}-\alpha_2}{(1+\frac{1+\alpha_2}{1+\alpha_1})^2}V\right)$$

$$= \frac{(1+\frac{1+\alpha_2}{1+\alpha_1})^2(-\frac{1+\alpha_2}{(1+\alpha_1)^2})-2(1+\frac{1+\alpha_2}{1+\alpha_1})(-\frac{1+\alpha_2}{(1+\alpha_1)^2})(\frac{1+\alpha_2}{1+\alpha_1}-\alpha_2)}{(1+\frac{1+\alpha_2}{1+\alpha_1})^4} = 0.$$

After some rearrangement it can be seen, that the nominator can only assume value 0, if

$$1 + \frac{1+\alpha_2}{1+\alpha_1} = 2\frac{1+\alpha_2}{1+\alpha_1} - 2\alpha_2$$

which is equivalent to

$$\text{(A1)} \qquad \alpha_1 = \frac{1+\alpha_2}{1+2\alpha_2} - 1 = -\frac{\alpha_2}{1+2\alpha_2}$$

(A1) gives the best reply function of player 1 as

$$\text{(B1)} \qquad \alpha_1(\alpha_2) = -\frac{\alpha_2}{1+2\alpha_2} \qquad \alpha_2 \in [-1, 1].$$

A symmetric argument w.r.t. player 2's maximization problem yields 2's best reply function as

$$\alpha_2(\alpha_1) = -\frac{\alpha_1}{1+2\alpha_1} \qquad \alpha_1 \in [-1, 1].$$

The following Lemma may then come as a surprise.

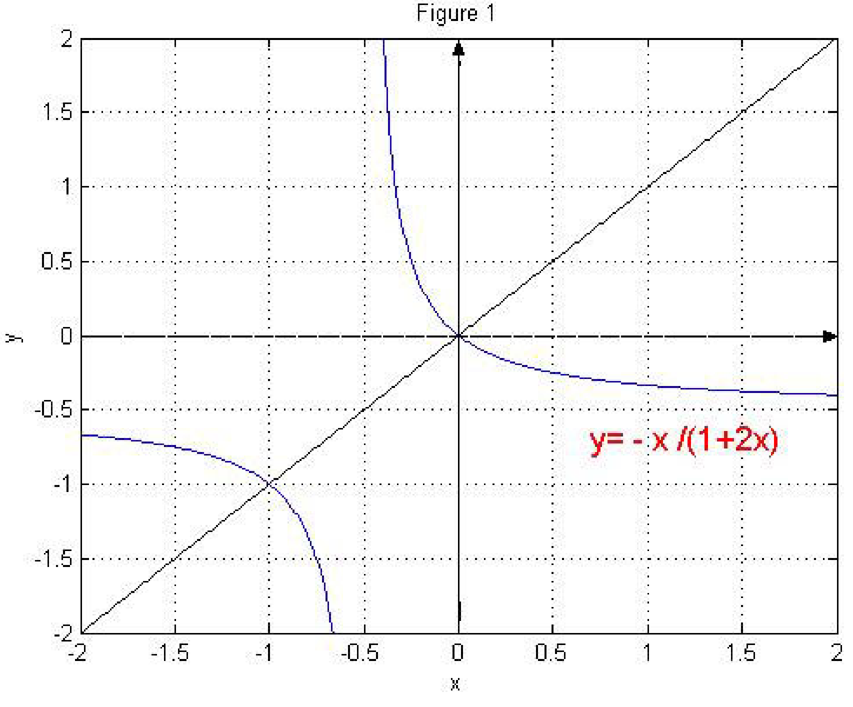
Lemma:
For any α, $\alpha \neq -\frac{1}{2}$, the combination $(\alpha, -\frac{\alpha}{1+2\alpha})$ is a Nash equilibrium.

Proof:
We have to show that $\alpha_2(\alpha_1(\alpha)) = \alpha$ for all $\alpha \neq -\frac{1}{2}$.

$$\begin{aligned} \alpha_2(\alpha_1(\alpha)) &= \alpha_2(-\frac{\alpha}{1+2\alpha}) = -\frac{\frac{-\alpha}{1+2\alpha}}{1+2(\frac{-\alpha}{1+2\alpha})} \\ &= \frac{\alpha}{1+2\alpha-2\alpha} = \alpha \quad \text{for all } \alpha \neq -\frac{1}{2} \end{aligned}$$

q.e.d.

In other words: Since the game is symmetric and yields symmetric best reply functions, those best reply functions *coincide* in (α_1, α_2)-space. Figure 1 illustrates this fact in detail.



We see from Figure 1, that only for values $\alpha_2 > -\frac{1}{3}$ the best reply of player 1 lies within his mutation space $M_1 = [-1, 1]$; the same holds for player 2 w.r.t. the type α_1 of player 1. Since we are not interested in the path of the evolutionary process, but only its possible rest points, we do not pursue and provide a full analysis, which would take account of boundary values when our restrictions on types would become binding. But it is obvious that the only relevant *symmetric* Nash equilibrium of the game is given by $(\alpha_1^*, \alpha_2^*) = (0, 0)$, which proves our Proposition 1, since this equilibrium is a strict one (which implies stability). The other symmetric solution of the first order condition; namely $\alpha_1 = \alpha_2 = -1$, is *not* an equilibrium w.r.t. $M = [-1, 1] \times [-1, 1]$. If e.g. 1 deviates to $\alpha_1 = -1 + \epsilon$, $\epsilon > 0$, then 2 would like to respond with an $\alpha_2 < -1$, but is prevented from doing so, which makes the deviation by 1 profitable.

It is worthwile to have a closer look at this family of Nash-equilibria.

Payoffs in equilibrium $(\alpha, \frac{\alpha}{1+2\alpha})$ are

$$F_1^*(\alpha, -\tfrac{\alpha}{1+2\alpha}) = \tfrac{1+2\alpha}{4(1+\alpha)}V$$

$$F_2^*(\alpha, -\tfrac{\alpha}{1+2\alpha}) = \tfrac{1}{4(1+\alpha)}V$$

Hence $F_1^* \geq F_2^* \Leftrightarrow \alpha \geq 0$.

Note that the sum of equilibrium payoffs is constant over all equilibria as

$$F_1^* + F_2^* = \tfrac{2+2\alpha}{4(1+\alpha)}V = \tfrac{1}{2}V.$$

It is only the *distribution* of fitness, which differs in different equilibria. Naturally, an unequal distribution would give a disadvantage to the recipient of the lower part. None of these equilibria can be evolutionarily stable. Note also, that only interdependent preferences of differing type can be equilibrium matches; if one is positively interdependent the other has to be negatively interdependent and vice versa.

Moreover, if we look at the 2-player contest $\bar{G}(\alpha, -\frac{\alpha}{1+2\alpha})$ we see that the Nash equilibrium bids are given by

$$x_1^*(\alpha, -\tfrac{\alpha}{1+2\alpha}) = \tfrac{1+2\alpha}{4(1+\alpha)}V \qquad \text{and}$$

$$x_2^*(\alpha, -\tfrac{\alpha}{1+2\alpha}) = \tfrac{1}{4(1+\alpha)}V \qquad \text{and hence}$$

$x_1^* + x_2^* = \frac{1}{2}V$ independent of α. In all contests, which are played with (Nash) *equilibrium* preferences of the evolutionary game, exactly one half of the value of the prize is dissipated in material terms, which leaves the other half of *material* payoff to be distributed between the two players in order to determine their fitness.

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