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Abstract

This paper revisits the excess entry theorem in spatial models à la Vickrey (1964) and Salop (1979) while relaxing the assumption of inelastic demand. Using a demand function with a constant demand elasticity, we show that the number of firms that enter a market decreases with the degree of demand elasticity. We find that the excess entry theorem does only hold when demand is sufficiently inelastic. Otherwise, there is insufficient entry. In the limiting case of unit elastic demand, the market is monopolized. We point out when and how a public policy can be desirable and broaden our results with a more general transportation cost function.

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1 Introduction

Three main frameworks have been widely used to study product differentiation and monopolistic competition: representative consumer, discrete choice and spatial models. In representative consumer and discrete choice models, it is understood that equilibrium product variety could either be excessive or insufficient or optimal depending on the model configuration.[1] In spatial models such as Vickrey (1964) and Salop (1979), however, analysis shows that there is always excessive entry. This result became known as the excess entry theorem. Matsumura and Okamura (2006) extend this result for a large set of transportation costs and production technologies.[2]

One drawback of standard spatial models such as Hotelling (1929) and Salop (1979) is that consumer demand is completely inelastic. Each consumer demands a single unit of a differentiated product.[3] The present paper lifts this restrictive assumption in the context of the Salop model and investigates the implications of price-dependent demand for the excess entry theorem.

To this aim, we incorporate a demand function with a constant elasticity into the Salop framework. We find that the number of entrants in a free-entry equilibrium is the lower the more elastic demand is. We also find that only when demand is sufficiently inelastic, there is excess entry. Otherwise, entry is insufficient. In the limiting case when the demand elasticity approaches unity, the market becomes a monopoly. Thus, the excess entry theorem is only valid for sufficiently inelastic demand and hence, the assumption of inelastic demand, typically employed, is not an innocuous one. As a consequence of our welfare analysis we point out when and how a public policy can be desirable. In an extension, we broaden our result with a more general transportation cost function.

Our model setup is closely related to Anderson and de Palma (2000). The purpose of their paper is to develop a model that integrates features of spatial models where competition is localized and representative consumer models where competition between firms is global. The formulation of the individual demand function is the same as in Anderson and de Palma (2000).[4] They also consider a constant elasticity demand function. However, the difference

[1]See, for example, Dixit and Stiglitz (1977), Pettengill (1979), Lancaster (1975), Sattinger (1984), Hart (1985) among many others.

[2]They do point out that there are also some situations in which entry can be insufficient.

[3]The assumption of inelastic demand can be a realistic one in the case of some durable goods, e.g. houses, etc. However, in case of nondurables, e.g. groceries, etc, the assumption seems less plausible.

[4]Our model is the special case of Anderson and de Palma (2000) when eliminating the taste component in their utility function. Thus, the present paper considers a pure spatial model, while Anderson and de Palma (2000) analyze a model that has features of spatial and representative consumer models.

lies in the perspective of the papers. Their focus is on the interaction between local and global competition, while the present paper focuses on the implications of price-dependent demand on the excess entry result in spatial models.

Other approaches to introduce price-dependent demand into spatial models are Boeckem (1994), Rath and Zhao (2001) and Peitz (2002).[5] The first two papers consider variants of the Hotelling framework. Boeckem (1994) introduces heterogenous consumers with respect to reservation prices. Depending on the price charged by firms some consumers choose not to buy a product. The paper by Rath and Zhao (2001) introduces elastic demand in the Hotelling framework by assuming that the quantity demanded by each consumer depends on the price charged. The authors propose a utility function that is quadratic in the quantity of the differentiated product leading to a linear demand function. In contrast to those two models we build on the Salop model as we are interested in the relationship between price-dependent demand and entry into the market. Our approach is closer to Rath and Zhao (2001) as we also assume that each consumer has a downward sloping demand for the differentiated good. However, our demand function takes on a different functional form which has the advantage of yielding tractable results. Peitz (2002) features unit-elastic demand both in Hotelling and Salop settings but focuses on conditions for existence of Nash equilibrium in prices. He does not consider entry decisions.

This paper is organized as follows. Section 2 sets up the model. Section 3 presents the analysis of the model. Section 4 analyzes the welfare outcome and policy implications. An extension with more general transportation cost functions is provided in section 5. Section 6 summarizes.

2 The model

There is a unit mass of consumers who are located on a circle with circumference one. The location of a consumers is denoted by x. In contrast to Salop (1979), consumers are not limited to buy a single unit of the differentiated good. The amount they purchase depends on the price. We propose the following utility function which leads to a demand function with a con-

[5] A recent paper by Peng and Tabuchi (2007) combines a model of spatial competition with taste for variety in the spirit of Dixit and Stiglitz (1977). In their setup, the quantity demanded also depends on the price. However, their focus is a different one. They study the incentives of how much variety to offer and how many stores to establish. A paper by Hamilton et al. (1994) analyzes elastic demand in a model with quantity competition. In contrast to the present note the authors employ a transportation costs per unit of quantity purchased.

stant elasticity of ϵ. We assume that this utility function is identical for all consumers:

$$U = \begin{cases} \left(V - \frac{\epsilon}{1-\epsilon} q_d^{\frac{\epsilon-1}{\epsilon}} - t * dist\right) + q_h & \text{if consumes the differentiated product} \\ q_h & \text{otherwise.} \end{cases} \tag{1}$$

The utility derived by the consumption of the differentiated good consists of three parts. There is a gross utility for consuming this good (V). The second utility component depends on the quantity consumed (q_d). The parameter ϵ —which lies between (0,1)—will later turn out to be the demand elasticity. Finally, consumers have to incur transportation costs if the product's attributes do not match consumers' locations. We assume that transportation costs do not depend on the quantity consumed. Furthermore, we assume that transportation costs are linear in distance.[6] In section 5, we will lift this assumption and cover a broader class of transportation cost functions, namely power transportation costs. The variable q_h denotes the quantity of a homogenous good which serves as a numeraire good. The utility is linear in this commodity. Additionally, we make the assumption that the gross utility of the differentiated good (V) is large enough such that no consumers abstains from buying the differentiated product.[7]

Each consumer has an exogenous income of Y which he can divide between the consumption of the differentiated good and the numeraire good. The price of the differentiated good is p_d, while the price of the numeraire is normalized to one. This leads to the following budget constraint:

$$Y = p_d * q_d + q_h. \tag{2}$$

Consumers maximize their utility (1) under their budget constraint (2). Then, demand for the differentiated product and the numeraire is:

$$\hat{q_d} = p_d^{-\epsilon}, \tag{3}$$

$$\hat{q_h} = Y - p_d^{1-\epsilon}. \tag{4}$$

[6] This allows a direct comparison to Salop (1979) model because the transportation cost is linear in that paper as well.

[7] This helps us to avoid situations in which a firm could be a local monopoly, hence the kink in the firm's demand curve.

The demand for the differentiated good exhibits a constant demand elasticity of ϵ. A higher value of ϵ corresponds to more elastic demand. The limit case of $\epsilon \to 0$ corresponds to completely inelastic demand. Inserting these demand functions into equation (1) gives the indirect utility a consumer derives from consuming the differentiated product from a certain firm:

$$\hat{U} = V + Y - \frac{1}{1-\epsilon} p_d^{1-\epsilon} - t * dist. \tag{5}$$

There are n firms that offer the differentiated product. We assume that these firms are located equidistantly on the circle. Hence, the distance between two neighboring firms is $\frac{1}{n}$. Consumers choose to buy the differentiated product from the firm which offers them the highest utility. Given the symmetric structure of the model, we seek for a symmetric equilibrium. Therefore we derive demand of a representative firm i. The marginal consumer is the consumer who is indifferent between choosing firm i and an adjacent firm. When firm i charges a price p_i while the remaining firms charge a price p, the marginal consumer is implicitly given by

$$V + Y - \frac{1}{1-\epsilon} p_i^{1-\epsilon} - t\bar{x} = V + Y - \frac{1}{1-\epsilon} p^{1-\epsilon} - t\left(\frac{1}{n} - \bar{x}\right), \tag{6}$$

or explicitly by

$$\bar{x} = \frac{1}{2n} + \frac{p^{1-\epsilon} - p_i^{1-\epsilon}}{2(1-\epsilon)t}. \tag{7}$$

As each firm faces two adjacent firms, the number of consumers choosing to buy from firm i is $2\bar{x}$. According to equation (3), each consumer buys an amount of $\hat{q}_i = p_i^{-\epsilon}$. Hence total demand at firm i is:

$$D_i = 2\bar{x} * p_i^{-\epsilon}. \tag{8}$$

In contrast to the Salop model, total demand consists now of two parts: market share and quantity per consumer.

3 Analysis

This section analyzes the equilibrium. We start by deriving equilibrium prices for a given number of firms in the market. In a second step, we seek to determine the number of firms that enter.

3.1 Price equilibrium

We look for a symmetric equilibrium in which all firms charge the same price. Assuming zero production costs, the profit of a representative firm i when this firm charges a price p_i and all remaining firms charge a price p is given by:

$$\Pi_i = \left[\frac{1}{n} + \frac{p^{1-\epsilon} - p_i^{1-\epsilon}}{(1-\epsilon)t}\right] p_i^{-\epsilon} p_i. \tag{9}$$

Maximizing profits with respect to the price p_i and assuming symmetry among all firms leads to the following equilibrium price:[8]

$$p^* = \left[(1-\epsilon)\frac{t}{n}\right]^{\frac{1}{1-\epsilon}}. \tag{10}$$

The corresponding quantity purchased by each consumer then is

$$q^* = \left[(1-\epsilon)\frac{t}{n}\right]^{-\frac{\epsilon}{1-\epsilon}}. \tag{11}$$

As in the Salop model, the equilibrium price increases in transportation costs and decreases in the number of firms in the market. Conversely, the quantity purchased by each consumer rises with the number of firms and decreases with transportation costs. More interesting is the impact of the demand elasticity on the equilibrium price and quantity. Differentiation with respect to ϵ yields:

$$\frac{\partial p^*}{\partial \epsilon} \gtreqless 0 \Leftrightarrow \frac{(1-\epsilon)t}{n} \gtreqless e, \tag{12}$$

$$\frac{\partial q^*}{\partial \epsilon} \lesseqgtr 0 \Leftrightarrow \frac{(1-\epsilon)t}{n} \gtreqless e^{\epsilon}. \tag{13}$$

where e denotes the Euler number. A higher demand elasticity has an ambiguous impact on equilibrium price and quantity. It can lead to a higher price as well as to a lower price. The intuition behind this result lies in the fact that firms can attract additional demand in two ways, via a larger market share and a larger quantity per consumer. Note, however, that the

[8] For the proof of the existence of a symmetric price equilibrium, the reader is referred to Anderson and de Palma (2000).

revenue per customer $p^*q^* = \frac{(1-\epsilon)t}{n}$ decreases in the price elasticity. In the limiting case of $\epsilon \to 1$, revenue per customer approaches zero.

In the equilibrium with a given number of firms in the market, each firm makes a profit of

$$\Pi^* = \frac{t(1-\epsilon)}{n^2}. \tag{14}$$

The impact of the demand elasticity on firms' profits is unambiguous. A larger demand elasticity reduces profits. This is due to the result that revenue per customer decreases with the demand elasticity and that the market share is constant at $\frac{1}{n}$ in equilibrium. Hence, product market competition is tougher as consumers react stronger to price changes. Higher transportation costs and a smaller number of active firms increase profits.

Result 1 *For a given number of firms, profits decrease with the demand elasticity.*

3.2 Entry

Until now the analysis treated the number of firms which offer differentiated products as exogenously given. We now investigate the number of active firms when it is endogenously determined by the zero profit condition. We assume that to enter, a firm has to incur an entry cost or fixed cost of f. Additionally, we treat the number of entrants as a continuous variable. Setting equation (14) equal to f and solving for n yields the number of entrants:

$$n^c = \sqrt{\frac{t(1-\epsilon)}{f}}. \tag{15}$$

The comparative static results concerning transportation costs and fixed costs are as expected. Higher transportation costs lead to more entry while higher fixed costs to less entry. The interesting result concerns the impact of the demand elasticity:

Result 2 *The number of entrants decreases in the demand elasticity.*

A larger demand elasticity leads to less entry into the market. The reason for this result is that a higher elasticity leads to lower profits for any given number of firms (see result 1).

Corresponding price and quantity in a free-entry equilibrium are:

$$p^c = \left[\sqrt{1-\epsilon}\sqrt{tf}\right]^{\frac{1}{1-\epsilon}}, \tag{16}$$

$$q^c = \left[\sqrt{1-\epsilon}\sqrt{tf}\right]^{-\frac{\epsilon}{1-\epsilon}}. \tag{17}$$

Higher transportation costs and higher fixed costs lead to higher prices and to lower quantities. As in the equilibrium for a given number of firms, the impact of the demand elasticity on price and quantity is ambiguous. More elastic demand may lead to higher or lower prices and quantities.

The model has interesting results in the limiting cases.

Result 3 *i) With $\epsilon \to 0$, the model reduces to the Salop model. ii) As $\epsilon \to 1$, the market is monopolized.*

When demand is completely inelastic, $\epsilon \to 0$, the model reduces to the Salop model. Thus that model is a special case of the present one. As the demand elasticity approaches unity, a monopoly is the outcome. Competition in the market is so tough that as soon as more than one firm enters the market profits are driven to zero (see equation (14)).

4 Welfare

This section considers the welfare and policy implications. We ask whether there is excess entry into the market as it is the case in models with inelastic demand.

In contrast to models with inelastic demand, we have to consider prices in our welfare analysis as they have an impact on the quantity purchased and hence on welfare. We define social welfare as the sum of consumer utility and industry profits:

$$W = \underbrace{V + Y - \frac{1}{1-\epsilon}p^{1-\epsilon} - 2n\int_0^{\frac{1}{2n}} tx\ dx}_{\text{Consumer welfare}} + \underbrace{p^{1-\epsilon} - fn}_{\text{Industry profits}}. \tag{18}$$

We determine social welfare given that prices are chosen by firms as given in the price equilibrium by equation (10). Inserting these prices gives

$$W = V + Y - \frac{t}{n} - 2n \int_0^{\frac{1}{2n}} tx \; dx + \frac{t(1-\epsilon)}{n} - fn. \qquad (19)$$

4.1 Socially optimal entry

Maximizing total welfare (19) with respect to n yields the optimal number of firms:[9]

$$n^w = \sqrt{\frac{t(1+4\epsilon)}{4f}}. \qquad (20)$$

Comparing the optimal number of firms, n^w, with the outcome under free entry, n^c, the following result can be established:

Result 4 *When $\epsilon < \frac{3}{8}$ there is excess entry. When $\epsilon > \frac{3}{8}$ there is insufficient entry. When $\epsilon = \frac{3}{8}$ entry is optimal.*

The previous result shows that the result of excess entry in the Salop model does not hold when demand is elastic. In the model with elastic demand whether there is too much entry or not enough depends on the demand elasticity. Whenever demand is sufficiently inelastic, there is excess entry as is the case in the Salop model ($\epsilon \to 0$). However, if the demand elasticity exceeds $\frac{3}{8}$, there is insufficient entry into the market. Only when $\epsilon = \frac{3}{8}$, entry coincides with the socially optimal number. Thus, the excess entry theorem in spatial models depends crucially on the assumption of inelastic demand.

4.2 Policy implications

Here we derive some policy implications of our welfare analysis. Suppose that a government agency may either charge a fee against or grant a subsidy to each entry, e.g. license fee or start-up funds, respectively. Let s denote the value of such a transfer. When $s < 0$ we call it a subsidy, and when $s > 0$ we call it an entry fee.

Hence the number of firms under such an otherwise "Free Entry" policy now is:

$$n^{c'} = \sqrt{\frac{t(1-\epsilon)}{f+s}}. \qquad (21)$$

[9]The second-order condition for maximization is satisfied: $-\frac{t(1+4\epsilon)}{2n^3} < 0$.

This, of course, follows directly from equation (15) by adjusting the fixed cost term accordingly. By setting equation (21) equal to (20), we can determine the value of s that induces optimal entry into the market. This value is

$$s = f\frac{3-8\epsilon}{1+4\epsilon}. \tag{22}$$

The following result summarizes the policy implication.

Result 5 *i) When $\epsilon < \frac{3}{8}$, a government agency should charge an entry fee to reduce excess entry; ii) when $\epsilon > \frac{3}{8}$, a government agency should subsidize entry.*

By such a transfer scheme, a government agency could effectively influence the number of active firms.

5 Power transportation costs

This section reconsiders the analysis assuming a more general transportation cost function. Instead of linear transportation costs, we now assume power transportation costs tx^{β} with $\beta \geq 1$. This functional form is also considered by Anderson et al. (1992) and Matsumura and Okamura (2006) which both show that the excess entry theorem always holds in the case of inelastic demand.[10] Our analysis will show that their result depends very much on the assumption of inelastic demand.

Following the same steps as in section 3, we can derive the number of entrants in a free-entry equilibrium and the socially optimal number. The derivation of these results is given in appendix A.

The number of entrants in a free-entry equilibrium is

$$n^c = \left[\frac{(1-\epsilon)t\beta 2^{1-\beta}}{f}\right]^{\frac{1}{1+\beta}}, \tag{23}$$

and the optimal number of firms is

$$n^w = \left[\frac{t\beta 2^{-\beta}(2\beta\epsilon + \frac{1}{1+\beta})}{f}\right]^{\frac{1}{1+\beta}}. \tag{24}$$

[10]Note that existence of price equilibrium is not ensured if β is too high. See Anderson et al. (1992, Ch. 6).

We denote by $\bar{\epsilon} = \frac{1+2\beta}{2(1+\beta)^2}$ the demand elasticity such that optimal and competitive entry coincides. This leads to the following result:

Result 6 *Suppose that transportation costs are of the power function form tx^{β}. Then we have that i) there is excess entry if $\epsilon < \bar{\epsilon}(\beta)$ and insufficient entry if $\epsilon > \bar{\epsilon}(\beta)$, and ii) $\bar{\epsilon}(\beta)$ decreases in β.*

The first part of the result generalizes result 4 for the case of a more general transportation cost function. It states that as long as demand is sufficiently inelastic the excess entry theorem still holds. Otherwise it does not hold. The second part of the result, follows directly as $\frac{\partial \bar{\epsilon}}{\partial \beta} = -\frac{\beta}{(1+\beta)^3} < 0$. It states that the interval of demand elasticities for which the excess entry theorem holds shrinks with β.

6 Conclusion

The present paper introduces elastic demand in the Salop (1979) model and investigates if the excess entry theorem still holds. We propose a utility function that leads to a demand function with constant elasticity. We find that a larger demand elasticity leads to less entry into the market. This is a hypothesis that can be tested empirically. Markets with higher demand elasticity should offer less product variety. In the limiting case of a unit demand elasticity the market outcome is a monopoly. Turning to welfare analysis, we show that when demand is sufficiently inelastic there is excess entry. However, when demand is sufficiently elastic the number of entrants is lower than the socially optimal number. Further, we provide conditions on when and how a government intervention can be desirable. We also show that our results hold with more general transportation cost functions.

A Derivation with power transportation costs

Here we provide the derivation of the results for the model with power transportation costs. The derivation follows Anderson et al. (1992, Ch. 6), but extended to price-dependent demand.

With power transportation costs, the marginal consumer is implicitly given by

$$-\frac{1}{1-\epsilon} p_i^{1-\epsilon} - t\bar{x}^{\beta} = -\frac{1}{1-\epsilon} p^{1-\epsilon} - t\left(\frac{1}{n} - \bar{x}\right)^{\beta}. \tag{25}$$

In contrast to the case of linear transportation costs, it is not possible to give a closed form for the marginal consumer. However, by total differentiation it is possible to calculate the impact of a price change on the marginal consumer, which is

$$\frac{d\bar{x}}{dp_i} = -\frac{p_i^{-\epsilon}}{t\beta(\bar{x}^{\beta-1} + (\frac{1}{n} - \bar{x})^{\beta-1})}. \tag{26}$$

As we are interested in a symmetric equilibrium we can evaluate this expression at the symmetric equilibrium, that is at $\bar{x} = \frac{1}{2n}$. Then, we get

$$\frac{d\bar{x}}{dp_i}\Big|_{\bar{x}=\frac{1}{2n}} = -\frac{p_i^{-\epsilon}}{2t\beta(\frac{1}{2n})^{\beta-1}}. \tag{27}$$

Profits for the representative firm i is $\Pi_i = 2\bar{x}p_i^{1-\epsilon}$. The first-order condition for profit maximization and assuming symmetry gives the following equilibrium prices for a given number of firms in the market:

$$p = \left[(1-\epsilon)\frac{t\beta 2^{1-\beta}}{n^{\beta}}\right]^{\frac{1}{1-\epsilon}}. \tag{28}$$

For $\beta = 1$, this gives the results of our base model, and for $\epsilon = 0$, we get the results of Anderson et al. (1992, Ch. 6). Each firm earns a profit of

$$\frac{(1-\epsilon)t\beta 2^{1-\beta}}{n^{\beta+1}} - f. \tag{29}$$

The number of firms that enter in a free-entry equilibrium is determined via the zero-profit condition. This leads to the following number of entrants:

$$n^c = \left[\frac{(1-\epsilon)t\beta 2^{1-\beta}}{f}\right]^{\frac{1}{1+\beta}}. \tag{30}$$

With power transportation costs total welfare can be expressed as:

$$W = V + Y - \frac{t\beta 2^{1-\beta}}{n^{\beta}} - \frac{t}{(1+\beta)n^{\beta}2^{\beta}} + \frac{(1-\epsilon)t\beta 2^{1-\beta}}{n^{\beta}} - fn. \tag{31}$$

The number of firms that maximizes total welfare is then

$$n^w = \left[\frac{t\beta 2^{-\beta}(2\beta\epsilon + \frac{1}{1+\beta})}{f} \right]^{\frac{1}{1+\beta}} . \quad (32)$$

Comparison with the number of firms in a free-entry equilibrium shows that there is excess entry if $\epsilon < \frac{1+2\beta}{2(1+\beta)^2}$.

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