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On the Restrictiveness of Separability

The Significance of Energy in German Manufacturing

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On the Restrictiveness of Separability: The Significance of Energy in German Manufacturing

Abstract
Any researcher would certainly agree with Hamermesh's (1993:34) intuition about separability that the ease of substitution between any two production factors should be unaffected by a third factor that is separable from the others. This paper emphasizes that such a notion of separability needs to be more restrictive than the classical separability concept is. We thus coin the notion of strict separability that implies the classical concept. By applying both separability concepts in a translog approach to German manufacturing data (1978–1990), we focus on the empirical question of whether the omission of energy affects the conclusions about the ease of substitution among non-energy factors. We find ample empirical evidence to doubt the assumption that energy is separable from all other production factors even in the relatively mild form of classical separability. At least under separability aspects, therefore, energy appears to be an indispensable production factor

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1 Introduction

Substitutability and separability are pivotal economic concepts in both utility and production theory and are indispensable for the understanding of the macroeconomic impacts of, for instance, energy price shocks. Although energy was frequently regarded as a negligible production factor before the 1970s' energy crises, since then a growing number of empirical studies has analyzed these issues. BERNDT and WOOD (1975, 1979), GRIFFIN and GREGORY (1976), and PINDYCK (1979) are seminal studies, with the cost shares of energy varying between 1 % and 10 % – for a survey, see e. g. FRONDEL and SCHMIDT (2002).

Whether or not a non-negligible production factor, such as energy after the oil crises, is omitted from any substitution analysis might be irrelevant for inferences about the ease of substitution between non-energy inputs if energy is separable from all other factors according to HAMERMESH's (1993:34) intuition: Given separability, substitution elasticity estimates of non-energy inputs should turn out to be the same irrespective of whether or not energy is included in the analysis. Obviously, such a notion of separability that explicitly refers to the invariance of substitution measures is intimately related to questions of choosing the correct model specification and particularly important when the data do not provide information on any non-negligible input factor, but do focus on the substitution relations of observable factors.

This paper investigates both theoretical and empirical aspects of the concept of separability and provides a clarification of the rigid nature of separability assumptions that guarantee the price invariance of substitution measures like cross-price elasticities. In our theoretical analysis, we highlight that, in dual approaches, classical separability of the factors i and j from a third factor k means that their input proportion x_i/x_j is unaffected by price changes in factor k. Yet, characterizing substitution relationships between two factors via input proportions is rather unusual in empirical studies, where the ease of substitution is typically measured by AES or MES, the ALLEN or MORISHIMA elasticities of substitution, and/or cross-price elasticities.

Therefore, we conceive a more restrictive notion that we call *strict separability*. This concept incorporates the classical definition of separability in that separability of the factors i and j from a factor k according to the classical notion is a necessary, but not sufficient, requirement for strict separability. Using German manufacturing data (1978-1990) and the prominent translog approach, we then empirically examine whether any separability assumptions hold so that the factor energy may be omitted from the database without affecting the cross-price elasticities among capital, labor, and materials. The reason for this examination is that several prior empirical studies investigating the issue of factor substitution for Germany involuntarily omitted energy due to the lack of data – see e. g. KUGLER *et al.* (1989).

The following section discusses the classical notion of separability, introduced independently by LEONTIEF (1947) and SONO (1945), and presents our more restrictive definition of separability. In Section 3, both the classical and our separability definition are applied to translog approaches. Separability test results for German manufacturing are reported in Section 4 and indicate that the factor energy should be included when analyzing substitution issues. Section 5 summarizes and concludes.

2 Separability and Substitution

Along the lines of, for example, BERNDT and CHRISTENSEN (1973:405), and based on a twice differentiable cost function $C(Y, p_1, p_2, ..., p_n)$ with non-vanishing first and second partial derivatives, we define the two factors i and j to be *separable*[1] from factor k if and only if the classical LEONTIEF-SONO separability condition is valid:

$$\frac{\partial}{\partial p_k}\left(\frac{\frac{\partial C(Y,p_1,\ldots,p_n)}{\partial p_i}}{\frac{\partial C(Y,p_1,\ldots,p_n)}{\partial p_j}}\right) = 0 \qquad \Longleftrightarrow \qquad \frac{\partial^2 C}{\partial p_i \partial p_k}\frac{\partial C}{\partial p_j} - \frac{\partial^2 C}{\partial p_j \partial p_k}\frac{\partial C}{\partial p_i} = 0. \qquad (1)$$

This standard concept of separability was employed by STROTZ (1957) to ana-

[1]Contrary to the literature, we do not distinguish strong from weak classical separability, a terminology coined by STROTZ (1957), because the intuition regarding substitution issues is exactly the same behind both concepts.

lyze two-stage optimization in utility theory: If classical separability assumptions hold, commodities can be divided into separable subsets, and intensities can first be optimized within each separable subset. Then, optimal intensities can be attained by holding fixed the within-subset intensities and optimizing the between-subset intensities.

When it comes to substitution issues, the natural intuition of separability is that the ease of substitution between any two production factors i and j should be unaffected by a factor k if k is separable from i and j. In fact, if the two inputs i and j are separable from factor k in classical terms, the *input proportion* x_i/x_j is independent of changes in factor k's price p_k, since – on the basis of SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i} = x_i$ – the classical separability condition (1) equals

$$\frac{\partial}{\partial p_k}\left(\frac{x_i(p_1, p_2, ..., p_n)}{x_j(p_1, p_2, ..., p_n)}\right) = 0. \tag{2}$$

Yet, in virtually no empirical study is the ease of substitution between i and j measured in terms of their *input proportion* x_i/x_j. Rather, empirical substitution studies employ cross-price elasticities, AES, or MES. The classical LEONTIEF-SONO condition (1), however, implies that neither AES, MES, nor cross-price elasticities

$$\eta_{x_i p_j} := \frac{\partial \ln x_i}{\partial \ln p_j} \qquad \text{and} \qquad \eta_{x_j p_i} := \frac{\partial \ln x_j}{\partial \ln p_i} \tag{3}$$

are unaffected by changes in the price of factor k.

Instead, the invariance of cross-price elasticities $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ due to changes in the price p_k is only guaranteed by the following definition of *strict separability*, for which we demonstrate that it implies the classical notion of separability. We define[2] the two factors i and j to be *strictly separable* from factor k if and only if

$$\frac{\partial}{\partial p_k}\eta_{x_i p_j} = 0 \qquad \text{and} \qquad \frac{\partial}{\partial p_k}\eta_{x_j p_i} = 0. \tag{4}$$

[2] Although there was some choice of specific approach, we decided to build our separability definition (4) on the basis of cross-price elasticities, because alternative definitions based on AES or MES would be even more restrictive than our definition, because both cross-price and own-price elasticities $\eta_{x_i p_j}$ and $\eta_{x_j p_j}$ are the basic ingredients of AES and MES – see, for instance, FRONDEL (2003).

Using SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i} = x_i$, and the definitions of $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ given in (3), our separability definition (4) may be written equivalently as

$$\frac{\partial^2 C}{\partial p_i \partial p_k} \frac{\partial^2 C}{\partial p_i \partial p_j} \Big/ \frac{\partial C}{\partial p_i} = \frac{\partial^3 C}{\partial p_i \partial p_j \partial p_k} \quad \text{and} \quad \frac{\partial^2 C}{\partial p_j \partial p_k} \frac{\partial^2 C}{\partial p_i \partial p_j} \Big/ \frac{\partial C}{\partial p_j} = \frac{\partial^3 C}{\partial p_i \partial p_j \partial p_k}. \qquad (5)$$

By equating both conditions, it can readily be seen that these conditions imply the classical LEONTIEF-SONO separability condition (1). In other words, the assumption of classical separability of i and j from k is a necessary, but not sufficient, condition for strict separability to hold and thus represents a weaker requirement than strict separability.

As a consequence, omitting a non-negligible factor, such as energy, from any substitution analysis might be unjustified even when the classical LEONTIEF-SONO separability conditions are satisfied, as it is not guaranteed that cross-price elasticities of non-energy factors are immune to price changes in the missing factor energy. This price immunity can only be ascertained if strict separability of energy from non-energy inputs holds. The concept of strict separability is exemplified in Sections 3 and 4 by the prominent and frequently employed translog approach – recent translog contributions are, for instance, YATCHEW (2000), RYAN and WALES (2000), and FILIPPINI (2001) – and a concrete application to German manufacturing data.

3 Separability and Translog Approaches

Translog cost functions are typically of the following structure – see CHRISTENSEN *et al.* (1971:255):

$$\ln C = \ln \beta_0 + \beta_Y \cdot \ln Y + \sum_{i \in F} \beta_i \cdot \ln p_i + \frac{1}{2} \sum_{i,j \in F} \beta_{ij} \ln p_i \ln p_j + \sum_{i \in F} \beta_{iT} \ln p_i \cdot T, \qquad (6)$$

where Y is a given level of output, F denotes a set of inputs, T a linear time trend that is included to capture technological progress, and symmetry of β_{ij} is imposed *a priori*. Under the assumption of optimal behavior and perfect competition, the cost share s_i of any factor $i \in F$ is given by

$$s_i = \frac{\partial \ln C}{\partial \ln p_i} = \beta_i + \beta_{iT} T + \sum_{l \in F} \beta_{il} \ln p_l \tag{7}$$

and the cross-price elasticity $\eta_{x_i p_j}$ reads – see e. g. FRONDEL and SCHMIDT (2006:188):

$$\eta_{x_i p_j} = \frac{\beta_{ij}}{s_i} + s_j. \tag{8}$$

The classical LEONTIEF-SONO separability condition (1) can be written equivalently as

$$\frac{\partial}{\partial p_k}\left(\frac{\frac{\partial \ln C}{\partial p_i}}{\frac{\partial \ln C}{\partial p_j}}\right) = 0 \iff \frac{\partial^2 \ln C}{\partial \ln p_i \partial \ln p_k}\frac{\partial \ln C}{\partial \ln p_j} - \frac{\partial^2 \ln C}{\partial \ln p_j \partial \ln p_k}\frac{\partial \ln C}{\partial \ln p_i} = 0. \tag{9}$$

Applied to translog cost function (6), this condition implies that the two factors i and j are separable from factor k if and only if

$$\beta_j \beta_{ik} - \beta_i \beta_{jk} + (\beta_{jT}\beta_{ik} - \beta_{iT}\beta_{jk}) \cdot T + \sum_{l \in F} (\beta_{jl}\beta_{ik} - \beta_{il}\beta_{jk}) \cdot \ln p_k = 0, \tag{10}$$

that is, if and only if

$$s_j \beta_{ik} - s_i \beta_{jk} = 0, \tag{11}$$

where s_i and s_j denote the cost shares of factor i and j, respectively.

For given factors $i, j,$ and k, equation (10) holds for all prices p_k and for any point of time T if and only if the following set of *nonlinear separability conditions* is satisfied:

$$\beta_j \beta_{ik} - \beta_i \beta_{jk} = 0,\ \beta_{jT}\beta_{ik} - \beta_{iT}\beta_{jk} = 0,\ \beta_{jl}\beta_{ik} - \beta_{il}\beta_{jk} = 0,\ l \in F. \tag{12}$$

For $l = k$, of course, $\beta_{jl}\beta_{ik} - \beta_{il}\beta_{jk} = 0$ is tautological. Obviously, system (12), as well as equation (11), is always satisfied if BERNDT and WOOD's (1975:266) so-called *linear* separability conditions hold:

$$\beta_{ik} = \beta_{jk} = 0. \tag{13}$$

These conditions are sufficient, but not necessary, for equation (11) and the nonlinear separability conditions (12) to hold. Thus, DENNY and FUSS (1977:404) are perfectly right in claiming that the linear separability conditions "are more restrictive than is readily apparent".

Strict separability of i and j from k defines that the cross-price elasticities $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ do not depend upon changes in the price p_k of factor k. Using expression (8) for cross-price elasticity $\eta_{x_i p_j}$, this definition implies, first,

$$0 = \frac{\partial}{\partial p_k}(\eta_{x_i p_j}) = \frac{\partial}{\partial p_k}(\frac{\beta_{ij}}{s_i} + s_j) = -\frac{\beta_{ij}}{s_i^2}\frac{\partial s_i}{\partial p_k} + \frac{\partial s_j}{\partial p_k}, \tag{14}$$

which is given for all concrete values of cost share s_i and parameter β_{ij} if

$$\frac{\partial s_i}{\partial p_k} = 0 = \frac{\partial s_j}{\partial p_k}, \tag{15}$$

that is, if both shares s_i and s_j are invariant to changes in the price p_k of factor k. For symmetry reasons, these properties also secure the second condition for strict separability of i and j from k, the invariance of $\eta_{x_j p_i}$ with respect to changes in price p_k.

On the basis of the cost-share expression (7), it can be easily seen that the cost shares s_i and s_j do not change with respect to p_k if the respective second-order coefficients β_{ik} and β_{jk} are equal to zero, that is, if the linear separability conditions (13) are fulfilled[3]. That is, the conditions for strict separability are identical to the linear separability conditions (13), which place severe restrictions on the functional form of the translog functions (DENNY and FUSS 1977:404). We argue, however, that only strict separability of i and j from factor k may conserve the cross-price elasticities $\eta_{x_j p_i}$ and $\eta_{x_i p_j}$ when there are substantial changes in the price of k. This invariance is particularly desirable if factor k needs to be omitted from the analysis due to data problems.

In the next section, we will address the specific issue of whether or not energy (E) can safely be omitted when analyzing substitution relations between the non-energy inputs capital (K), labor (L), and materials (M) in German manufacturing and whether or not changes of energy prices would alter the cross-price elasticities of non-energy inputs. To this end, we employ both our more restrictive separability definition (4) and the classical LEONTIEF-SONO conditions of separability (1). We argue that strict separability would have to be tested even if classical separability of E from $K, L,$ and M did hold. Yet, conversely, the violation of the classical separability conditions implies

[3]Note that in the special case of a COBB-DOUGLAS technology, for which $\beta_{ij} = 0$ for all i and j, strict separability is always given for all input factors.

that energy is not strictly separable from all other inputs and should already put the issue at rest. Finally, we investigate if there is a difference in cross-price elasticities of non-energy inputs when energy is included in the analysis versus when it is excluded.

4 Empirical Evidence for German Manufacturing

Because of data limitations due to the German reunification, our data base merely includes the short range of 1978-1990. Overall, we have $377 = 29 \times 13$ observations originating from 29 German manufacturing sectors. The data necessary for estimation include cost shares and price and quantity indices for our set of production factors, $F = \{K, L, E, M\}$. The data sources and methods for constructing price and quantity series are described in Appendix A.

We assume that the twice-differentiable *aggregate* translog cost function (6) is appropriate for all industries of German manufacturing. In other words, we pool all the data for the 29 sectors in order to estimate the parameters of a common translog cost function. Linear homogeneity in prices, an inherent feature of any cost function, requires the following conditions:

$$\beta_K + \beta_L + \beta_E + \beta_M = 1\,, \tag{16}$$

$$\beta_{Kl} + \beta_{Ll} + \beta_{El} + \beta_{Ml} = 0 \qquad \text{for all } l \in F = \{K, L, E, M\}, \tag{17}$$

$$\beta_{KT} + \beta_{LT} + \beta_{ET} + \beta_{MT} = 0\,. \tag{18}$$

Unknown parameters might be estimated directly from a stochastic version of (6), but resulting estimates are well-known to show large standard errors. Yet, it is widely known in the econometric literature that efficiency gains can be realized by estimating a system of cost-share equations – see e. g. BERNDT (1991:470). In our example, the stochastic version of the cost-share equation system reads as follows:

$$s_K = \beta_K + \beta_{KK}\ln(\frac{p_K}{p_M}) + \beta_{KL}\ln(\frac{p_L}{p_M}) + \beta_{KE}\ln(\frac{p_E}{p_M}) + \beta_{KT}\cdot T + \varepsilon_K$$

$$s_L = \beta_L + \beta_{KL} \ln(\frac{p_K}{p_M}) + \beta_{LL} \ln(\frac{p_L}{p_M}) + \beta_{LE} \ln(\frac{p_E}{p_M}) + \beta_{LT} \cdot T + \varepsilon_L, \tag{19}$$

$$s_E = \beta_E + \beta_{KE} \ln(\frac{p_K}{p_M}) + \beta_{LE} \ln(\frac{p_L}{p_M}) + \beta_{EE} \ln(\frac{p_E}{p_M}) + \beta_{ET} \cdot T + \varepsilon_E \ ,$$

where the restrictions (17) and (18) are already imposed and disturbances are denoted by $\varepsilon_K, \varepsilon_L$, and ε_E. In order to avoid the singularity of the disturbance covariance matrix, because cost shares always add to unity, the share equation for materials (M) has been dropped arbitrarily. The unknown parameters of the seemingly unrelated regression (SUR) model (19) are preferably estimated using maximum likelihood (ML) methods to ensure that results do not depend upon the choice of which share equation is dropped (BERNDT 1991:473).

Testing *classical* $[(K, L, M), E])$ and $[(K, L), (M, E)]$ separability consists of two parts: Upon the rejection of the sufficient, but not necessary, linear conditions

$$\beta_{KE} = \beta_{LE} = \beta_{ME} = 0 \tag{20}$$

and, respectively,

$$\beta_{KE} = \beta_{LE} = \beta_{KM} = \beta_{LM} = 0, \tag{21}$$

the corresponding nonlinear classical separability conditions may still hold and need to be examined in order to check whether or not classical separability is given. Yet, if (20) and (21) are already invalid, *strict* $[(K, L, M), E])$ and $[(K, L), (M, E)]$ separability is violated.

It is derived in Appendix B that for classical $[(K, L, M), E]$ separability the following 7 *nonlinear* restrictions are necessary and sufficient:

$$\beta_L = (\beta_E - 1)\frac{\beta_{KL}}{\beta_{KE}},\ \beta_{LL} = \frac{\beta_{KL}^2}{\beta_{KE}^2} \cdot \beta_{EE},\ \beta_{LE} = \frac{\beta_{KL}}{\beta_{KE}} \cdot \beta_{EE},\ \beta_{LT} = \frac{\beta_{KL}}{\beta_{KE}} \cdot \beta_{ET}, \tag{22}$$

$$\beta_K = (\beta_E - 1)\frac{\beta_{KE}}{\beta_{EE}},\ \beta_{KK} = \frac{\beta_{KE}^2}{\beta_{EE}},\ \beta_{KT} = \frac{\beta_{KE}}{\beta_{EE}} \cdot \beta_{ET} \ . \tag{23}$$

Note that, if the linear conditions (20) are rejected, $\beta_{KE} \neq 0$ and, furthermore, $\beta_{EE} = -(\beta_{KE} + \beta_{LE} + \beta_{ME})$ may equal zero only by chance. Classical $[(K, L), (M, E)]$ separability merely requires 4 *nonlinear* restrictions:

$$\beta_L = \beta_K \frac{\beta_{KL}}{\beta_{KK}},\ \beta_{LL} = \frac{\beta_{KL}^2}{\beta_{KK}},\ \beta_{LE} = \frac{\beta_{KL}}{\beta_{KK}} \cdot \beta_{KE},\ \beta_{LT} = \frac{\beta_{KL}}{\beta_{KK}} \cdot \beta_{KT} \ . \tag{24}$$

Note that replacing β_K, β_{KK}, and β_{KT} in (24) with the terms given in (23) reproduces (22).

The classical $[(K, L), (M, E)]$ separability results reported in Table 1 cast doubt on prior value-added studies, since they indicate that the separability conditions required for value-added approaches are violated. With particular respect to the separability of energy from non-energy inputs, classical and, consequently, strict $[(K, L, M), E]$ separability must be rejected at all significance levels. These results call into question those prior empirical studies for German manufacturing that have abstained from the factor energy. Estimates of cross-price elasticities for K, L, and M provided by those studies can hardly be expected to be reliable. Of course, this also holds for AES and MES, which build on cross-price elasticities.

Table 1: Separability Tests – German Manufacturing (1978 - 1990).

Kind of Separability Tests	Degrees of Freedom	Test Results
linear separability conditions (20), (21)		
$[(K, L, M), E]$	3	32.5^{**}
$[(K, L), (M, E)]$	4	40.0^{**}
nonlinear separability conditions (22), (23), (24)		
$[(K, L, M), E]$	7	67.3^{**}
$[(K, L), (M, E)]$	4	35.9^{**}

Note: ** denotes significance at the 1 % level.

One of the reasons for this conclusion is that inappropriately imposing the linear $[(K, L, M), E]$ separability conditions (20), $\beta_{KE} = \beta_{LE} = \beta_{ME} = 0$, on cost-share system (19) would cause mis-specification:

$$\begin{aligned} s_K &= \beta_K + \beta_{KK} \ln(\frac{p_K}{p_M}) + \beta_{KL} \ln(\frac{p_L}{p_M}) + 0 \cdot \ln(\frac{p_E}{p_M}) + \beta_{KT} \cdot T + \varepsilon_K \\ s_L &= \beta_L + \beta_{KL} \ln(\frac{p_K}{p_M}) + \beta_{LL} \ln(\frac{p_L}{p_M}) + 0 \cdot \ln(\frac{p_E}{p_M}) + \beta_{LT} \cdot T + \varepsilon_L, \\ s_E &= \beta_E + 0 \cdot \ln(\frac{p_K}{p_M}) + 0 \cdot \ln(\frac{p_L}{p_M}) + 0 \cdot \ln(\frac{p_E}{p_M}) + \beta_{ET} \cdot T + \varepsilon_E \,. \end{aligned} \tag{25}$$

On the other hand, if strict separability of energy from non-energy inputs were to hold true, energy-related variables would not appear in the first and the second equation of the cost-share system (25) and only the third equation would comprise energy-related parameters. The appearance of the degenerated system (25) has quite an intuitive appeal: At first glance, it seems as if system (25) may be separated into two parts that can be estimated separately, with the first part consisting of the first and second equation and the second part encompassing the third equation of (25). If this were true, one could easily refrain from the inclusion of the variable energy and the lack of energy data would not be problematic at all when interest is on the estimation of cross-price elasticities among the non-energy inputs capital, labor, and materials.

Yet, there are two aspects that destroy this intuition. First, the degenerated system (25) is a SUR model that requires simultaneous estimation procedures rather than estimation part by part or equation by equation. Second, cost shares of non-energy inputs would be different and clearly higher than otherwise if a non-negligible factor – in terms of cost shares – such as energy were to be ignored in the analysis. For both these reasons, the omission of energy may well be consequential for the cross-price elasticity estimates of non-energy inputs even if strict separability of energy from non-energy inputs is actually given, that is, even if the degenerated system (25) were the correct specification.

This is because, without having energy data in hand, one is forced to estimate a reduced cost-share system such as

$$\begin{aligned} s_K &= \beta_K + \beta_{KK}\ln(\frac{p_K}{p_M}) + \beta_{KL}\ln(\frac{p_L}{p_M}) + \beta_{KT}\cdot T + \varepsilon_K, \\ s_L &= \beta_L + \beta_{KL}\ln(\frac{p_K}{p_M}) + \beta_{LL}\ln(\frac{p_L}{p_M}) + \beta_{LT}\cdot T + \varepsilon_L. \end{aligned} \tag{26}$$

In other words, in addition to the strict separability conditions (20), $\beta_{KE} = \beta_{LE} = \beta_{ME} = 0$, one is then obliged to further invoke $\beta_E = 0$ and $\beta_{ET} = 0$ in the degenerated system (25). Yet, imposing these five conditions on the lower equation of (25) would only be justified if the cost share of energy, s_E, were negligible. In short, even if strict $[(K, L, M), E]$ separability were to hold true, an exclusion of the factor energy from the analysis might change the estimates of non-energy cross-price elasticities, if this factor is non-negligible.

To finally investigate the issue of whether or not cross-price elasticities of non-energy inputs remain the same when the factor energy is dropped from our KLEM data set, we have deliberately caused mis-specification and have estimated the reduced cost-share system (26). We have found that cross-price elasticity estimates change only moderately after omitting energy from the data base. This outcome is in line with FRONDEL and SCHMIDT's (2002) cost-share argument that, in static translog approaches, estimates of cross-price elasticities $\eta_{x_i p j}$ are mainly the result of the corresponding cost shares s_j, because cost shares of K, L, and M remain almost unchanged when ignoring energy with its commonly low cost shares[4]. But rather than rehabilitating prior German KLM studies, these results cast doubt on static translog approaches and support FRONDEL and SCHMIDT's (2002:72) somewhat pessimistic message that "[s]tatic translog approaches are limited in their ability to detect a wide range of phenomena".

5 Summary and Conclusion

With particular respect to substitution issues, the natural intuition of the two factors i and j being separable from a factor k is that k should not affect the ease of substitution among the other two factors[5] – see e. g. HAMERMESH (1993:34). The classical definition of separability of factor i and j from factor k implies that in dual approaches their input proportion x_i/x_j is unaffected by the price of factor k. However, the overwhelming majority of empirical substitution studies analyzes the ease of substitution between two factors on the basis of cross-price elasticities, AES, or MES, rather than by input

[4]Similarly, SHARP, BLAIR, and WATKINS (1987:365) find that the recognition of intermediate materials M as a separate factor appears to be more important than separating energy factors such as electricity. Certainly, this has to do with the typically large cost shares of M.

[5]There are obvious similarities to SAMUELSON's (1974) famous paradox, in which the ease of substitution between coffee and cream depends on whether or not cross-price effects between coffee and cream are calculated holding the quantity of the third good tea constant. OGAKI (1990) resolves SAMUELSON's paradox by defining the concepts of direct and indirect substitutes and focusing on the direct substitution effect between coffee and cream with the quantity of tea held constant as the price of coffee falls (WEBER 2002:278).

proportions.

In this paper, we have therefore suggested a more restrictive notion of separability that we call strict separability. We define two factors i and j to be *strictly separable* from factor k if and only if both cross-price elasticities, $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$, remain unaffected by changes in the price of factor k. We have demonstrated that our concept of strict separability incorporates the classical notion of separability. Applied to dual translog approaches, we have shown that strict separability of i and j from factor k holds if and only if the linear separability conditions for the second-order coefficients of the translog cost function are valid:

$$\beta_{ik} = \beta_{jk} = 0.$$

These conditions are known to be sufficient, but not necessary, for classical separability and according to DENNY and FUSS (1977) are more restrictive than necessary. However, we argue that only *these* restrictive conditions capture a notion of separability of the factors i and j from k that guarantees the invariance of substitution possibilities among i and j – measured in terms of cross-price elasticities – when the price of factor k varies substantially. This property is particularly helpful if factor k must be omitted from the analysis – for instance, due to the lack of data.

In a concrete application of both separability concepts to German manufacturing data (1978-1990), we find that classical and, hence, strict $[(K, L, M), E]$ as well as $[(K, L), (M, E)]$ separability must be rejected. These results cast doubt on prior KLM studies for German manufacturing. Furthermore, our theoretical considerations for translog approaches indicate that even if energy were separable from non-energy inputs in the sense of our restrictive concept of strict separability, one cannot hope that cross-price elasticities of non-energy inputs would remain unaffected by the exclusion of energy in any substitution analysis.

Therefore, we conclude that there appears to be no separability concept that satisfies HAMERMESH's intuition about the invariance of the ease of substitution among two factors i and j being separable from a third factor k. In sum, while it is natural that energy is taken into account in studies aiming at the macroeconomic impacts of drastic rises of energy prices, the inclusion of the factor energy seems to be indispensable even when estimating substitution possibilities between non-energy inputs.

Appendix A Data

Data necessary for estimation include cost shares, price indices and quantity indices for K, L, E and M. These data originate from two separate sources – Input-Output Tables and National Accounts –, because data on energy are not available in the National Accounts. Energy expenditures and quantities based on the Input-Output classification (1978-90, unpublished data) have been provided by the Federal Statistical Office. We use this information for splitting up gross materials into energy and (non-energy) materials. Data from both sources are not directly comparable, though. For this reason, we are forced to use the same adjustments described by FALK and KOEBEL (1999) to make energy data based on Input-Output Tables consistent with data stemming from National Accounts. Because of data limitations for energy data, our data base relates to the short range of 1978-1990. Overall, we have 377 $= 29 \times 13$ observations from $S = 29$ sectors of German manufacturing. Unfortunately, for 3 of a total of 32 sectors of German manufacturing, not all data necessary are available.

Cost shares

Labor cost shares s_L are the sum of wages and salaries paid yearly in each industry in relation to gross production values generated in the corresponding industries. Capital cost shares s_K are the differences between gross value added and labor cost shares of each industry. Energy cost shares s_E are each industry's energy expenditures related to its gross production value. Cost shares for M result from the differences between gross production values and energy cost shares. For the energy-intensive industries of German Manufacturing, cost shares are displayed in Table A1.

Quantity and Price Indices

Dividing labor cost by the average number of employees in each industry yields the average price of labor for each year and, by normalizing to one in 1978, the corresponding price indices p_L for labor. Capital K is the net capital stock at 1985 prices. Then, capital price indices are obtained by dividing capital cost, which is the residual of gross value added and labor cost, by K and normalization to one in 1978. Energy price indices are constructed similarly on the basis of energy cost and energy quantities E (in terajoule), both given by the Input-Output Tables. Finally, real gross production values, that is gross production values at constant prices, are calculated with the help of producer price indices. Then, quantities for M are constructed by subtracting real gross value added from real gross production values. The deflator for (non-energy) materials p_M is calculated by dividing materials expenditures by their respective quantities.

Appendix B Nonlinear Separability Conditions

Classical Nonlinear $[(K, L), (M, E)]$ **separability constraints**

The necessary and sufficient conditions derived in Section 4 for classical separability of two factors $i = K$ and $j = L$ from k yield exactly the same set of conditions for both $k = M$ and $k = E$:

$$\frac{\beta_L}{\beta_K} = \frac{\beta_{KL}}{\beta_{KK}} = \frac{\beta_{LL}}{\beta_{KL}} = \frac{\beta_{LE}}{\beta_{KE}} = \frac{\beta_{LM}}{\beta_{KM}} = \frac{\beta_{LT}}{\beta_{KT}} . \tag{27}$$

These five restrictions are equivalent to the set of four equations (24), as one condition in (27) is superfluous when the linear homogeneity restrictions (17) and the first three equations of (27) are applied:

$$\frac{\beta_{LM}}{\beta_{KM}} = \frac{\beta_{LK} + \beta_{LL} + \beta_{LE}}{\beta_{KK} + \beta_{KL} + \beta_{KE}} = \frac{\frac{\beta_L}{\beta_K}\beta_{KK} + \frac{\beta_L}{\beta_K}\beta_{KL} + \frac{\beta_L}{\beta_K}\beta_{KE}}{\beta_{KK} + \beta_{KL} + \beta_{KE}} = \frac{\beta_L}{\beta_K} . \tag{28}$$

Classical Nonlinear $[(K, L, M), E]$ **separability constraints**

In addition to the set of restrictions given by (27), which is obtained for $i = K, j = L$, and $k = E$, classical $[(K, L, M), E]$ separability requires

$$\frac{\beta_M}{\beta_K} = \frac{\beta_{KM}}{\beta_{KK}} = \frac{\beta_{LM}}{\beta_{KL}} = \frac{\beta_{ME}}{\beta_{KE}} = \frac{\beta_{MM}}{\beta_{KM}} = \frac{\beta_{MT}}{\beta_{KT}} . \tag{29}$$

These conditions result from the set of nonlinear conditions (12) when $i = K, j = M$, and $k = E$. For $i = L, j = M$, and $k = E$, one cannot gain further information beyond that already given by (27) and (29).

Similar to (27), one of the five conditions in (29) is superfluous due to linear homogeneity requirements. Moreover, equation

$$\frac{\beta_{KM}}{\beta_{KK}} = \frac{\beta_{LM}}{\beta_{KL}} \qquad \Longleftrightarrow \qquad \frac{\beta_{LM}}{\beta_{KM}} = \frac{\beta_{KL}}{\beta_{KK}} \tag{30}$$

is already contained in (27) so that, in fact, we are left with 7 independent $[(K, L, M), E]$-separability constraints. These 7 restrictions reduce the initial number of 12 independent parameters occurring in system (19) to 5: $\beta_E, \beta_{KE}, \beta_{EE}, \beta_{ET}$ and β_{KL}.

In order to derive the set of seven equations (22) - (23), displayed in Section 4, we depart from (27) and use the three conditions in the middle of (27) and the linear homogeneity restrictions (17):

$$\beta_{LE} \cdot \beta_{MK} = \beta_{KE} \cdot \beta_{LM} = -\beta_{KE}(\beta_{LL} + \beta_{LE} + \beta_{LK}) = -\beta_{KE}(\frac{\beta_{KL}^2}{\beta_{KK}} + \frac{\beta_{KL}}{\beta_{KK}}\beta_{KE} + \beta_{LK}) \; . \tag{31}$$

Combining $\frac{\beta_{LE}}{\beta_{KL}} = \frac{\beta_{KE}}{\beta_{KK}}$ and $\frac{\beta_{KE}}{\beta_{KK}} = \frac{\beta_{ME}}{\beta_{MK}}$ and using (17) again, yields

$$\beta_{LE} \cdot \beta_{MK} = \beta_{ME} \cdot \beta_{KL} = -\beta_{KL}(\beta_{EE} + \beta_{KE} + \beta_{LE}) = -\beta_{KL}(\beta_{EE} + \beta_{KE} + \frac{\beta_{KL}}{\beta_{KK}}\beta_{KE}) \; . \tag{32}$$

By equating (31) and (32), we have the second constraint of (23),

$$\beta_{KK} = \frac{\beta_{KE}^2}{\beta_{EE}} \; . \tag{33}$$

Next, when using (16) and (17), the first condition in (29),

$$\frac{\beta_M}{\beta_K} = \frac{\beta_{KM}}{\beta_{KK}} \qquad \Longleftrightarrow \qquad \frac{1 - \beta_K - \beta_L - \beta_E}{\beta_K} = -\frac{\beta_{KK} + \beta_{KL} + \beta_{KE}}{\beta_{KK}}, \tag{34}$$

is equivalent to

$$\beta_K = (\beta_E - 1)\frac{\beta_{KK}}{\beta_{KE}} \; , \tag{35}$$

when $\frac{\beta_L}{\beta_K} = \frac{\beta_{KL}}{\beta_{KK}}$ is applied. By expression (33), this is the same as the first condition in (23),

$$\beta_K = (\beta_E - 1)\frac{\beta_{KE}}{\beta_{EE}} \; . \tag{36}$$

By considering $\frac{\beta_L}{\beta_K} = \frac{\beta_{KL}}{\beta_{KK}}, \frac{\beta_{LT}}{\beta_{KT}} = \frac{\beta_{KL}}{\beta_{KK}}$, and (36), the following equation of (29),

$$\frac{\beta_M}{\beta_K} = \frac{\beta_{MT}}{\beta_{KT}} \qquad \Longleftrightarrow \qquad \frac{1 - \beta_E - \beta_K - \beta_L}{\beta_K} = -\frac{\beta_{KT} + \beta_{LT} + \beta_{ET}}{\beta_{KT}} \; , \tag{37}$$

leads to the third condition of (23),

$$\beta_{KT} = \frac{\beta_{KE}}{\beta_{EE}} \cdot \beta_{ET} \; . \tag{38}$$

Finally, the expressions (36), (33) and (38) for β_K, β_{KK} and β_{KT}, respectively, substituted in the set of four nonlinear conditions (24) for $[(K, L), (M, E)]$ separability yield the remaining four nonlinear constraints (22) for $[(K, L, M), E]$-separability.

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